UNIVERSITY FOR DEVELOPMENT STUDIES

A CONSTRUCTION OF P-GROUPS VIA WREATH PRODUCTS

BY

ALHASSAN ELVIS ADAM (BSc. MATHEMATICS)

(UDS/MM/0001/08)

Thesis submitted to the Department of Mathematics, Faculty of Mathematical Sciences, University for Development studies in partial fulfillment of the requirements for the award of Master of Science Degree in Mathematics

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2011

Declaration

Student

I hereby declare that this thesis is the result of my own original work and that no part of it has been presented for another degree in this University or elsewhere.

Candidate's Signature: Date: 1st December, 2011

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Supervisors'

I hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University for Development Studies.

Principal Supervisor's Signature: Date: 1st December, 2011

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Name: Oladejo N. K.

Abstract

This research is essentially an upgrade or extension of the results of Audu on wreath product of permutation groups.

In this work, a generalization of the concept of wreath product is considered based on the notion of an algebraic structure called Permutation group. Some basic procedures for computing wreath products of groups, the construction of groups as wreath products of cyclic groups and a generation of Sylow p-subgroups from resulting Cyclic groups are investigated.

I mainly applied the concept of p-groups and wreath products of some groups of permutations to construct some groups of prime power order by taking p to be an arbitrary but fixed prime number, Δ a finite set with n elements and G a permutation group on Δ.

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To all I say, God bless you.

Dedication

dedicate this piece of work to my dad and mum Mr. and Mrs. Alhassan for their love and care

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CHAPTER ONE

1.1 Background

Recently, wreath product of groups has been used to explore some useful characteristics of finite groups in connection with permutation designs and construction of lattices as well as in the study of interconnection networks and the structure of some finite permutation groups.

This research also provides a theoretical frame work for generating various Sylow subgroups from permutations in cyclic forms.

1.2 Problem Statement and Justification

Wreath product of groups first occurred in a not immediately recognizable form in a paper by Alfred Loewy (http://thoughtmesh.net/publish/396.php) and since then, it has become an increasingly important tool in group theory. However, very little work of wreath product in p-groups has been researched. It is against this background that this research is carried out to provide baseline information for further research activities in the study area.

1.3 Objectives of the study

The main objective of this study is to construct p-groups via wreath products

Specific Objectives:

- To write out some basic procedures for computing wreath product of groups
- To construct groups as wreath products of cyclic groups
- To generate sylow p-subgroups from resulting cyclic groups

1.4 Scope of the study

The study is in four chapters. The first is the background of the study. Chapter two is a review of related literature whiles chapter three consists of the main results and findings and chapter four discusses the results and findings, conclusion and recommendations.

1.5 Definition of terms

1.5.1 Definition 1.1

A set G equipped with a binary operation say * is called a groupoid denoted by (G. *) or just G.

1.5.2 Definition 1.2

A groupoid (G, *) such that * is associative is called a semi-group.

1.5.3 Definition 1.3

A semi-group (G, *) with an identity element is called a monoid.

1.5.4 Definition 1.4

A monoid (G, *) in which every element has an inverse is called a group.

1.5.5 Definition 1.5

A group is a p-group if the order is a prime or prime power.

1.5.6 Definition 1.6

Direct product of a group is the Cartesian product of the underlying groups with component-wise multiplication.

1.5.7 Definition 1.7

The semi direct product of a group H with a group G acting on H via homomorphism μ from G into the automorphism group of H is the Cartesian product G×H with the multiplication $(g, n).(h, m) = (gh, n^{h\mu}m)$.

In short, it is a particular way in which a group can be put together from two subgroups, one of which is a normal subgroup.

1.6 Preliminary Results

The following preliminary results will be required in the construction of p-groups as proposed in this research work.

1.6.1 Group Action

If a group G is acting on a subgroup H of G, then G is equipped with the restriction of the operation of H.

Let G be a group and Ω be a non-empty set. We say that G acts on Ω (or that G permutes Ω) if to each $g \ in \ G$ and each $\alpha \in \Omega$ there corresponds a unique point $\alpha g \in \Omega$ such that, for all $\alpha \in \Omega \ and \ g_1, g_2 \ in \ G \ we \ have \ that \ (\alpha g_1)g_2 = \alpha(g_1 g_2)$ and $\alpha.1 = \alpha$.

The image of an action is a permutation group.

For instance, if we let H and K be two groups. We say that H acts on K as a group if to each k in K there corresponds a unique element K^h in K such that for h_1, h_2, h in H and k_1, k_2, k in K

$$\Rightarrow (k^{h_1})^{h_2} = k^{h_1 h_2}, k^1 = k \text{ and } (k_1 k_2)^h = k_1^{\ h} k_2^{\ h}$$

1.6.2 Theorem 1.1

Let C and D be permutation groups on the sets Γ *and* Δ respectively. Let C^{Δ} be the set of all maps of Δ into the permutation group C. That is $C^{\Delta} = \{f : \Delta \to C\}$. . For any f_1, f_2 *in* C^{Δ}, let f_1, f_2 *in* C^{Δ} be defined for all δ *in* Δ *by* $(f_1 f_2)(\delta) = f_1(\delta) f_2(\delta)$.

Thus, composition of functions is point-wise. With respect to this operation of multiplication, C^{Δ} acquires the structure of a group.

Proof

(i) C^{Δ} is non-empty and is closed with respect to multiplication.

If f_1, f_2 *in* C^{Δ}, then $f_1(\delta), f_2(\delta)$ *in* C.

Hence, $f_1(\delta) f_2(\delta) \in C$. This implies that $(f_1 f_2) \in C$ and so $f_1 f_2 \in C^{\Delta}$

(ii) Since multiplication is associative so also is the multiplication in C^{Δ}

(iii) The identity element in C^{Δ} is the map $e : \Delta \to C$ given by $e(\delta) = 1$ *for all* $\delta \in \Delta$ *and* $1 \in C$.

(iv) Every element $f \in C^{\Delta}$ is defined for all $\delta \in \Delta$ by $f^{-1}(\delta) = f(\delta)^{-1}$.

Thus, C^{Δ} is a group with respect to the multiplication defined above.

(We denote this group by P)

1.6.3 Lemma 1.1

Assume that D acts on P as follows: $f^{d}(\delta)=f(\delta d^{-1})$ for all $\delta \in \Delta$, $d \in D$. Then D acts on P as a group.

Proof

Take $f, f_1, f_2 \in P$ and $d, d_1, d_2 \in D$ then

$$(i)\ (f^{d_1})^{d_2}(\delta)=f^{d_1}(\delta d_2^{-1})$$
$$=f(\delta d_2^{-1}d_1^{-1})$$
$$=f^{d_1 d_2}(\delta)$$

$$(ii)\ f^{1}(\delta)=f(\delta_1^{-1})$$
$$=f(\delta)$$

$$(iii)\ (f_1 f_2)^{d}(\delta)=f_1 f_2(\delta d^{-1})$$
$$=f_1(\delta d^{-1})f_2(\delta d^{-1})$$
$$=f_1^{\,d}(\delta)f_2^{\,d}(\delta)$$

Thus, D acts on P as a group

1.6.4 Theorem 1.2

Let D act on P as a group. Then the set of all ordered pairs (f,d) with $f \in P$ and $d \in D$ acquires the structure of a group when we define for all $f_1, f_2 \in P$ and $d_1, d_2 \in D$

$$(f_1,d_1)(f_2,d_2)=(f_1 f_2 d_1^{-1})(d_1 d_2)$$

Proof

(i) Closure property follows from the definition of multiplication

(ii) Take $f_1, f_2, f_3 \in P$ and $d_1, d_2, d_3 \in D$. Then

$$[(f_1,d_1)(f_2d_2)](f_3,d_3) = (f_1f_2d_1^{-1})(d_1d_2)(f_3,d_3)$$
$$= (f_1f_2^{d_1^{-1}}f_3^{(d_1d_2)^{-1}})(d_1d_2d_3)$$
$$= (f_1f_2^{d_1^{-1}}f_3^{(d_2^{-1}d_1^{-1})})(d_1d_2d_3)$$

Also, we have in the same manner that

$$(f_1,d_1)[(f_2d_2)(f_3,d_3)] = (f_1,d_1)(f_2f_3^{d_2^{-1}})(d_2d_3)$$
$$= (f_1(f_2f_3^{d_2^{-1}})^{d_1^{-1}})(d_1d_2d_3)$$
$$= (f_1f_2^{d_1^{-1}}f_3^{(d_2^{-1}d_1^{-1})})(d_1d_2d_3)$$

Hence, multiplication is associative

(iii) We know that for every $f \in P$, $f^1 = f$. Now for every $d \in D$, the map $f \to f^d$ is an automorphism of P. Also if e is the identity element in P, then $e^d = e$. Also, $(f^{-1})^d = (f^d)^{-1}$. Now

$(f,d)(e,1) = (fe^{d^{-1}}, d.1) = (fe^{d^{-1}}, d) = (f(e^{-1}), d) = (f,d)$

Also, using the reverse order, we have that; $(e,1)(f,d) = (ef^{1^{-1}}, 1d) = (ef,d) = (f,d)$

Thus, the identity element exists

(iv) $(f,d)((f^{-1})^d, d^{-1}) = (f(f^{-1})^d, dd^{-1}) = (f(f^{-1})dd^{-1}, dd^{-1}) = (f(f^{-1})^1, dd^{-1} = (e,1)$

Also,

$((f^{-1})^d, d^{-1})(f,d) = ((f^{-1})^d f^d, d^{-1}d) = (ff^{-1})^d, d^{-1}d) = (e^d,1) = (e,1)$

Thus, when D acts on P, the set of all ordered pairs (f,d) *with* $f \in D, d \in D$ is a group if we define $(f_1,d_1)(f_2,d_2) = (f_1f_2^{d_1^{-1}})(d_1d_2)$

CHAPTER TWO

2.0 Literature Review

2.1 Introduction

In Mathematics, wreath product of groups is a specialized product of two groups, based on a semi direct product. Wreath products are an important tool in the classification of Permutation groups and also provide a way of constructing interesting examples of groups.

2.2 According to Jaiyeola et al (2009) 'on some algebraic properties of generalized groups' a generalized group G exhibits the following algebraic properties:

$(i)\ For\ \ all\ \ x,y,z\in G,\ \ (xy)z=x(yz)$

$(ii)\ For\ each\ x\in G\ there\ exists\ a\ unique\ e(x)\in G\ such\ \ that\ xe(x)=e(x)x=x$
$(existence\ and\ uniqueness\ \ of\ \ identity\ element)$

$(iii)\ For\ each\ x\in G,\ there\ \ exists\ a\ unique\ x^{-1}\in G\ such\ that\ xx^{-1}=x^{-1}x=e(x)$
$(existence\ of\ \ inverse\ element)$

2.3 Molaei (2007)

Molaei (2007) in his work on generalized groups established the uniqueness of the identity element of each element in a generalized group and where the identity element is not unique for each element, then a group is formed.

Some of his results are stated below

2.3.1 Theorem 2.1

For each element x in a generalized group G, there exists a unique $x^{-1} \in G$.

Proof

Suppose $x \in G$,

$$then\ xe(x) = x(x^{-1}x) = x(x^{-1}e(x^{-1})x) = (x^{-1}e(x^{-1}))x = [xx^{-1}e(x^{-1})]x = (e(x)e(x^{-1}))x = x$$

and similarly, $x = x(e(x^{-1}))e(x)$ so the uniqueness of $e(x)$ shows that

$$e(x)e(x^{-1}) = e(x^{-1}) = e(x)$$

Hence, $e(x^{-1}) = e(x)$ ------------- (1)

Now let $zx = xz = e(x)$ *and* $yx = xy = e(x)$. then $z(xy) = z(e(x)),\ so\ e(x)y = ze(x)$

which shows that $y = z$ *from* (1) *above*

2.3.2 Theorem 2.2

Let a generalized group G act on a set S. Then for every

$x \in S,\ the\ set\ I_x = \{g \in G; gx = x\}\ is\ a\ generalized\ subgroup\ of\ G.$

Proof: For $g \in I_x$ we have:

$$gx = x \Rightarrow (e(g)g)x = x \Rightarrow e(g)(gx) = x$$
$$\Rightarrow e(g)x = x \Rightarrow e(g) \in I_x,$$

and $\quad g^{-1}x = g^{-1}(gx) = (g^{-1}g)x = e(g)x = x.$

So $\quad g^{-1} \in I_x.$

If $\quad g_1, g_2 \in I_x,$

then $\quad (g_1g_2)x = g_1(g_2x) = g_1x = x.$

Hence $\quad g_1g_2 \in I_x.$

Thus I_x is a generalized subgroup of G. $\square$

2.4 Cheryl E. Praeger and Csaba Schneider

In a paper titled "Ordered Triple Designs and Wreath Products of Groups", Cheryl E Praeger and Csaba Schneider obtained the following results:

Let Γ be a finite set, $L \leq Sym\Gamma$, $\ell \geq 2$ *an integer and* $H \leq S_\ell$. *the wreath product*

L wr H is the semi direct product

$L^\ell \times H$ *where for* $(x_1,\ldots,x_\ell) \in L\ell$ *and* $\sigma \in S_\ell$, $(x_1,\ldots,x_\ell)^\sigma = (x_{1\sigma^{-1}},\ldots,x_{\ell\sigma^{-1}})$.

The product action of L wr H is the action of L wr H on Γ^ℓ

defined by $(\gamma_1,\ldots,\gamma_\ell)(x_1,\ldots,x_\ell)\sigma = (\gamma_{1\sigma^{-1}}{}^{x_{1\sigma^{-1}}},\ldots,\gamma_{\ell\sigma^{-1}}{}^{x_{\ell\sigma^{-1}}})$

for all $(\gamma_1,\ldots,\gamma_\ell) \in \Gamma^\ell, (x_1,\ldots,x_\ell)\sigma \in L\,wr\,H$.

Thus, if $\gamma \in \Gamma$ *then* $(\gamma_1,\ldots,\gamma_\ell) \in \Gamma^\ell$, *set* $\omega = (\gamma,\ldots,\gamma)$,
the stabiliser $(L\,wr\,H)_\omega$ *in* $L\,wr\,H$ *of* ω *is the subgroup* $(L_\gamma)^\ell \times H = L_\gamma\; wr\,H$,
where L_γ *is a stabiliser of* γ *in* L. *(It is easy to see that* H *normalises* $(L_\gamma)^\ell$
and so $(L_\gamma)^\ell \times H$ *is indeed a subgroup of* $L\,wr\,H$).

The subgroup L^ℓ *is normal in* $L\,wr\,H$ *and is transitive on* Γ^ℓ *if and only if*
L *is transitive on* Γ

Moreover no non-identity element of L wr H stabilises all points of Γ^ℓ.

In other words, the product action of L wr H on Γ^ℓ is faithful. Therefore L wr H can be considered as a permutation group on Γ^ℓ.

2.5 Agboola (2002) indicated that if $\prod_{i=1}^{n} G_i$ be the direct product of generalized groups G_i and let $(g_1, g_2, ---, g_n)$ *and* $(h_1, h_2, ---, h_n)$ be in $\prod_{i=1}^{n} G_i$. Then $\left(\prod_{i=1}^{n} G_i, \otimes\right)$ is also a generalized group where $\otimes$ is a multiplicative binary operation defined by:

$$(g_1, g_2, ---, g_n) \otimes (h_1, h_2, ---, h_n) = (g_1h_1, g_2h_2, ---, g_nh_n) \;\; \forall\, g_i, h_i \in G_i, i = 1, 2, ---, n$$

Proof

It is clear that $\otimes$ is closed and associative over $\prod_{i=1}^{n} G_i$. Now for each $g_i \in G_i, e_{g_i} \in G_i$, is the unique identity with respect to g_i and so on

$$\begin{aligned}(g_1, g_2, ---, g_n) \otimes (e_{g_1}, e_{g_2}, ---, eg_n) &= (e_{g_1}, e_{g_2}, ---, e_{g_n}) \otimes (g_1, g_2, ---, g_n) \\ &= (g_1, g_2, ---, g_n)\end{aligned}$$

This shows that $(g_1, g_2, ---, g_n) \in \prod_{i=1}^{n} G_i$ is the unique identity

with respect to $(g_1, g_2, ---, g_n) \in \prod_{i=1}^{n} G_i$.

Lastly, for each $(g_1, g_2, ---, g_n) \in \prod_{i=1}^{n} G_i$, we have

$$\begin{aligned}(g_1, g_2, ---, g_n) \otimes (g_1^{-1}, g_2^{-1}, ---, g_n^{-1}) &= (g_1g_2^{-1}, g_2g_2^{-1}, ---, g_ng_n^{-1}) \\ &= (e_{g_1}, e_{g_2}, ---, e_{g_n})\end{aligned}$$

Using a similar argument, we have

$$(g_1^{-1}, g_2^{-1}, ---, g_n^{-1}) \otimes (g_1, g_2, ---, g_n) = (e_{g_1}, e_{g_2}, ---, e_{g_n})$$

This shows that $(g_1^{-1}, g_2^{-1}, ---, g_n^{-1})$ is unique inverse of $(g_1, g_2, ---, g_n) \in \prod_{i=1}^{n} G_i$.

Accordingly, $\left(\prod_{i=1}^{n} G_i, \otimes\right)$ is a generalized group.

2.6 Ibrahim A. A and Audu M. S (2007)

Ibrahim A. A and Audu M. S in a paper titled "Wreath Product of Permutation Groups" obtained the following results on Transitivity, Primitivity and faithfulness of W on $\Gamma \times \Delta$.

2.6.1 Transitivity of W on Γ x Δ

Suppose that we take two arbitrary points (α_1, δ_1) *and* (α_2, δ_2) *in* $\Gamma \times \Delta$.

$\forall\, \alpha_1, \alpha_2 \in \Gamma$ *and* $\delta_1, \delta_2 \in \Delta$, then W will be transitive on $\Gamma \times \Delta$ if there exists $fd \in W,\ \forall f \in P\, and\, d \in D$ such that $(\alpha_1, \delta_1)^{fd} = (\alpha_2, \delta_2)$. This will hold if $(\alpha_1 f(\delta_1), \delta_1 d) = (\alpha_2, \delta_2)$; *i.e. if* $\alpha_1 f(\delta_1) = \alpha_2, \delta_1 d = \delta_2$.

Thus, such fd exists if C and D are transitive on Γ and Δ respectively, which is the necessary condition for W to be transitive on ΓxΔ.

2.6.2 Faithfulness of W on ΓxΔ

We recall that W is faithful on ΓxΔ if the identity element of W is the only element that fixes every point of ΓxΔ. Now the identity element of W is 1 and thus if W is to be faithful on ΓxΔ, then for any (ά,δ) in ΓxΔ, the stabilizer of W on (ά,δ), $W_{(ά,δ)}$ must be $F(δ)_{ά}D_{δ} = 1$. Hence, $F(δ)_{ά} = 1$ and $D_{ά} = 1$ for all ά ϵ Γ, δ ϵ Δ. But áf(δ) = ά, δd = δ for all f(δ) ϵ $F(δ)_{ά}$ and for all d ϵ $D_{δ}$. This must imply that f(δ) = 1 and d = 1. Thus, we deduce that W would be faithful on $\Gamma x\Delta$ if the stabilizers of any $\acute{\alpha}\epsilon\Gamma$ and $\delta\epsilon\Delta$ are the identity elements in P and D respectively. Therefore, we conclude that W would be faithful on $\Gamma x\Delta$ if P or C and D are faithful on Γ and Δ respectively.

2.6.3 Primitivity of W on ΓxΔ

Recall that W would be primitive on $\Gamma x\Delta$ if and only if given any $(\acute{\alpha}, \delta)$ in $\Gamma x\Delta$, $W_{(\acute{\alpha},\delta)}$, the stabilizer of $(\acute{\alpha}, \delta)$ is a maximal subgroup of W. In what follows, we provide this necessary and sufficient condition for the Primitivity of W on $\Gamma x\Delta$.

Now $W_{(\alpha,\delta)} = F(\delta)_\alpha D_\delta$, where $F(\delta)_\alpha$ is the set of those f in P such that $f(\delta)$ fixes α, and D_δ is the stabilizer of δ under the action of D on Δ.

As $F(\delta)_\alpha$ does not include those f in P which do not stabilize α, we have that $F(\delta)_\alpha < P$.

We note that in general, $P < W$, $P D_\delta$ is a subgroup of W and it is proper unless $D_\delta = D$. We therefore conclude that

$W_{(\alpha,\delta)} = F(\delta)_\alpha D_\delta < P D_\delta < P D = W$

Hence $W_{(\alpha,\delta)}$ is not a maximal subgroup of W. Thus W would be imprimitive on $\Gamma \times \Delta$ in a natural way.

However, if $|\Gamma| = 1$; that is, $\Gamma = \{\alpha\}$ say, then $C_\Gamma = C_\alpha = C$.

In particular, $\alpha f(\delta) = \alpha$ for all f in P.

Thus, $F(\delta)_\alpha = P$ and hence $F(\delta)_\alpha D_\delta = P D_\delta$. And if in addition, D is primitive on Δ then D_δ is maximal in D and hence

$P D_\delta = F(\delta)_\alpha D_\delta = W(\alpha,\delta)$ will be maximal in W. That is, W will be primitive on $\Gamma \times \Delta$.

Again, if $|\Delta| = 1$; that is, $\Delta = \{\delta\}$ say, then $D_\delta = D$ and

$W_{(\alpha,\delta)} = F(\delta)_\alpha D_\delta = F(\delta) D$

And if in addition, C is primitive on Γ, then C_α will be maximal in

$C = \{f(\delta) \mid \delta \in \Delta, f \in P\}$. Correspondingly, $F(\delta)_\alpha$ will be maximal in P and hence $W_{(\alpha,\delta)}$ will be maximal in W. That is, W will be primitive on $\Gamma \times \Delta$.

In conclusion, we have shown that W is imprimitive on $\Gamma \times \Delta$ in a natural way, unless $|\Gamma| = 1$ and D is primitive on Δ or $|\Delta| = 1$ and C is primitive on Γ.

CHAPTER THREE

3.0 Results and Findings

3.1 Wreath Product

3.1.1 Definition 3.1

The wreath product of C by D denoted by W = CwrD is the semi-direct product of P by D, so that, $W = \{(f,d) | f \in P, d \in D\}$, *with multiplication in W defined as*

$$(f_1,d_1)(f_2,d_2) = ((f_1 f_2^{d_1^{-1}})(d_1 d_2)) \ \forall \ f_1, f_2 \in P \text{ and } d_1, d_2 \in D$$

Henceforth, we write fd instead of (f, d) for elements of W.

3.1.2 Theorem 3.1

Let D act on P as $f^d(\delta) = f(\delta d^{-1})$ *where* $f \in P$, $d \in D$ *and* $\delta \in \Delta$.

Let W be the group of all juxtaposed symbols fd, with $f \in P$, $d \in D$

and multiplication given by $(f_1,d_1)(f_2,d_2) = (f_1 f_2 d_1^{-1})(d_1 d_2)$.

Then W is a group called the semi-direct product of P by D with the defined action.

3.1.3 Remarks 3.1

- If C and D are finite groups, then the wreath product W determined by an action of D on a finite set is a finite group of order $|W| = |C|^{|\Delta|}.|D|$
- P is a normal subgroup of W and D is a subgroup of W
- The action of W on $\Gamma \times \Delta$ is given by

  $(\alpha, \beta) fd = (\alpha f(\beta), \beta d)$ *where* $\alpha \in \Gamma$ *and* $\beta \in \Delta$

3.2 Procedure for constructing wreath products of groups

The "wreath product" of two groups G and H is built as follows. Express H as a permutation group, permuting n items. Make n copies of G. Let H act on these n copies of G, permuting them. Thus, the wreath product of G by H is the semi direct product of (the direct product of n copies of G) by H.

Of course, the above procedure is undefined until we have specified how to express H as a permutation group. The brainless way to do this is to regard H as permuting the elements of H itself (the permutations being given by the Cayley's table of H). This is known as the "regular wreath product" of G by H. We will write this as G rwp H.

Another way is to express H as a permutation group of a small number of items, in as pretty a way as possible. For instance, C_6 can be expressed as (a b c d e f), or as (a b).(p q r): the former is prettier, but the latter uses fewer items.

In the special cases in which H is a symmetric group, "in as pretty a way as possible" *is* well defined: the symmetric group S_n should be expressed as permutations of n items. The cases where H is the symmetric group S_n and is expressed as permutations of n items give what is known as "permutation wreath products".

3.3 Main Results

The main results obtained in constructing some p-groups by means of wreath product of two permutation groups as introduced above are as follows:

3.3.1 Consider the Permutation Groups A_1 and B_1

$A_1 = \{(1), (1\ 2\ 3), (1\ 3\ 2)\}$ and $B_1 = \{(1), (4\ 5\ 6), (4\ 6\ 5)\}$ acting on the sets

$\Gamma_1 = \{1, 2, 3\}$ and $\Delta_1 = \{4, 5, 6\}$ respectively.

Let $P = A_1^{\Delta_1} = \{f : \Delta_1 \to A_1\}$. *For any* $f_1, f_2 \in A_1^{\Delta_1}$, *then* $|P| = |A_1|^{|\Delta_1|} = 3^3 = 27$

Thus, P will have 27 mappings as below:

$f_1 : 4 \to (1), 5 \to (1), 6 \to (1)$
$f_2 : 4 \to (123), 5 \to (123), 6 \to (123)$
$f_3 : 4 \to (132), 5 \to (132), 6 \to (132)$
$f_4 : 4 \to (1), 5 \to (123), 6 \to (132)$
$f_5 : 4 \to (1), 5 \to (132), 6 \to (123)$
$f_6 : 4 \to (123), 5 \to (1), 6 \to (132)$
$f_7 : 4 \to (123), 5 \to (132), 6 \to (1)$
$f_8 : 4 \to (132), 5 \to (123), 6 \to (1)$
$f_9 : 4 \to (132), 5 \to (1), 6 \to (123)$
$f_{10} : 4 \to (1), 5 \to (1), 6 \to (123)$
$f_{11} : 4 \to (1), 5 \to (1), 6 \to (132)$
$f_{12} : 4 \to (123), 5 \to (123), 6 \to (1)$
$f_{13} : 4 \to (123), 5 \to (123), 6 \to (132)$
$f_{14} : 4 \to (132), 5 \to (132), 6 \to (1)$
$f_{15} : 4 \to (132), 5 \to (132), 6 \to (123)$

$f_{16}: 4 \to (1),\ 5 \to (123),\ 6 \to (123)$
$f_{17}: 4 \to (132),\ 5 \to (123),\ 6 \to (123)$
$f_{18}: 4 \to (1),\ 5 \to (132),\ 6 \to (132)$
$f_{19}: 4 \to (123),\ 5 \to (132),\ 6 \to (132)$
$f_{20}: 4 \to (1),\ 5 \to (123),\ 6 \to (1)$
$f_{21}: 4 \to (1),\ 5 \to (132),\ 6 \to (1)$
$f_{22}: 4 \to (123),\ 5 \to (1),\ 6 \to (123)$
$f_{23}: 4 \to (123),\ 5 \to (132),\ 6 \to (123)$
$f_{24}: 4 \to (132),\ 5 \to (1),\ 6 \to (132)$
$f_{25}: 4 \to (132),\ 5 \to (123),\ 6 \to (132)$
$f_{26}: 4 \to (123),\ 5 \to (1),\ 6 \to (1)$
$f_{27}: 4 \to (132),\ 5 \to (1),\ 6 \to (1)$

It can be verified that P is a group with respect to the operations:

$(f_1 f_2)(\delta) = f_1(\delta) f_2(\delta)$, *where* $\delta \in \Delta_1$.
We recall the definition of the action of B_1 *on* P *as* $f^d(\delta_1) = f(\delta_1 d^{-1})$
Where $f \in P$, $d \in B_1$ *and* $\delta_1 \in \Delta_1$, *then* B_1 *acts on* P *as a group* (α_1, δ_1) *and* (α_2, δ_2) *in* $\Gamma_1 \times \Delta_1$.

We also recall the definition W = A_1 wr B_1, the semi-direct product of P by B_1 in that order, that is, $W = \{(f,d) | f \in P,\ d \in B_1\}$. Now W is a group with respect to the operation $(f_1 d_1)(f_2 d_2) = (f_1 f_2^{d_1^{-1}})(d_1 d_2)$, *and*

accordingly, $d_1 = (1)$, $d_2 = (456)$ *and* $d_3 = (465)$ *Then the elements of* W *are* :

$(f_1 d_1), (f_2 d_1), (f_3 d_1), (f_4 d_1), (f_5 d_1), (f_6 d_1), (f_7 d_1), (f_8 d_1), (f_9 d_1), (f_{10} d_1),$

$(f_{11} d_1), (f_{12} d_1), (f_{13} d_1), (f_{14} d_1), (f_{15} d_1), (f_{16} d_1), (f_{17} d_1), (f_{18} d_1), (f_{19} d_1), (f_{20} d_1),$
$(f_{21} d_1), (f_{22} d_1), (f_{23} d_1), (f_{24} d_1), (f_{25} d_1), (f_1 d_2), (f_2 d_2), (f_3 d_2), (f_4 d_2), (f_5 d_2),$
$(f_6 d_2), (f_7 d_2), (f_8 d_2), (f_9 d_2), (f_{10} d_2), (f_{11} d_2), (f_{12} d_2), (f_{13} d_2), (f_{14} d_2), (f_{15} d_2),$
$(f_{16} d_2), (f_{17} d_2), (f_{18} d_2), (f_{19} d_2), (f_{20} d_2), (f_{21} d_2), (f_{22} d_2), (f_{23} d_2), (f_{24} d_2), (f_{25} d_2)$
$(f_{26} d_1), (f_{27} d_1), (f_1 d_2), (f_2 d_2), (f_3 d_2), (f_4 d_2), (f_5 d_2), (f_6 d_2), (f_7 d_2), (f_8 d_2),$
$(f_9 d_2), (f_{10} d_2), (f_{11} d_2), (f_{12} d_2), (f_{13} d_2), (f_{14} d_2), (f_{15} d_2), (f_{16} d_2), (f_{17} d_2), (f_{18} d_2),$

$(f_{19}d_2),(f_{20}d_2),(f_{21}d_2),(f_{22}d_2),(f_{23}d_2),(f_{24}d_2),(f_{25}d_2),(f_{26}d_2),(f_{27}d_2)(f_1d_3),$
$(f_2d_3),(f_3d_3),(f_4d_3),(f_5d_3),(f_6d_3),(f_7d_3),(f_8d_3),(f_9d_3),(f_{10}d_3),(f_{11}d_3),$
$(f_{12}d_3),(f_{13}d_3),(f_{14}d_3),(f_{15}d_3),(f_{16}d_3),(f_{17}d_3),(f_{18}d_3),(f_{19}d_3),(f_{20}d_3),(f_{21}d_3),$
$(f_{22}d_3),(f_{23}d_3),(f_{24}d_3),(f_{25}d_3),(f_{26}d_3),(f_{27}d_3)$

Further, $\Gamma_1 \times \Delta_1 = (\beta,\delta) = \{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$
where $\Gamma_1 = \{1,2,3\}$ *and* $\Delta = \{4,5,6\}\ \forall\ \beta \in \Gamma_1, \delta \in \Delta_1$

Now define the action of W on $\Gamma_1 \times \Delta_1$ as

$(\beta,\delta)fd = (\beta f(\delta), d\delta)$ *where* $\beta \in \Gamma_1$ *and* $\delta \in \Delta_1$.

We obtain the following by the action of W on $\Gamma_1 \times \Delta_1$

$(1,4)f_1d_1 = (1f_1(4),\delta d_1) = (1(1),4(1)) = (1,4)$
$(1,5)f_1d_1 = (1f_1(5),\delta d_1) = (1(1),5(1)) = (1,5)$
$(1,6)f_1d_1 = (1f_1(6),\delta d_1) = (1(1),6(1)) = (1,6)$
$(2,4)f_1d_1 = (2f_1(4),\delta d_1) = (2(1),4(1)) = (2,4)$
$(2,5)f_1d_1 = (2f_1(5),\delta d_1) = (2(1),5(1)) = (2,5)$
$(2,6)f_1d_1 = (2f_1(6),\delta d_1) = (2(1),6(1)) = (2,6)$
$(3,4)f_1d_1 = (3f_1(4),\delta d_1) = (3(1),4(1)) = (3,4)$

$(3,5)f_1d_1 = (3f_1(5),\delta d_1) = (3(1),5(1)) = (3,5)$
$(3,6)f_1d_1 = (3f_1(6),\delta d_1) = (3(1),6(1)) = (3,6)$

and in summary, we form the permutations below:

$$(\Gamma_1 \times \Delta_1)f_1d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \end{pmatrix}$$
$$(\Gamma_1 \times \Delta_1)f_2d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,4),(2,5),(2,6),(3,4),(3,5),(3,6),(1,4),(1,5),(1,6) \end{pmatrix}$$
$$(\Gamma_1 \times \Delta_1)f_3d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,4),(3,5),(3,6),(1,4),(1,5),(1,6),(2,4),(2,5),(2,6) \end{pmatrix}$$
$$(\Gamma_1 \times \Delta_1)f_4d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,4),(2,5),(3,6),(2,4),(3,5),(1,6),(3,4),(1,5),(2,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_5 d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,4),(3,5),(2,6),(2,4),(1,5),(3,6),(3,4),(2,5),(1,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_6 d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,4),(1,5),(3,6),(3,4),(2,5),(1,6),(1,4),(3,5),(2,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_7 d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,4),(3,5),(1,6),(3,4),(1,5),(2,6),(1,4),(2,5),(3,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_8 d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,4),(2,5),(1,6),(1,4),(3,5),(2,6),(2,4),(1,5),(3,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_9 d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,4),(1,5),(2,6),(1,4),(2,5),(3,6),(2,4),(3,5),(1,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{10} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,4),(1,5),(2,6),(1,6),(2,4),(3,5),(3,6),(3,4),(2,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{11} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,4),(1,5),(3,6),(2,4),(2,5),(1,6),(3,4),(3,5),(2,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{12} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,4),(2,5),(1,6),(3,4),(3,5),(2,6),(1,4),(1,5),(3,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{13} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,4),(2,5),(3,6),(3,4),(3,5),(1,6),(1,4),(1,5),(2,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{14} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,4),(3,5),(1,6),(1,4),(1,5),(2,6),(2,4),(2,5),(3,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{15} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,4),(3,5),(2,6),(1,4),(1,5),(3,6),(2,4),(2,5),(1,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{16} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,4),(2,5),(2,6),(2,4),(3,5),(3,6),(3,4),(1,5),(1,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{17} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,4),(2,5),(2,6),(1,4),(3,5),(3,6),(2,4),(1,5),(1,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{18} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,4),(3,5),(3,6),(2,4),(1,5),(1,6),(3,4),(2,5),(2,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{19} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,4),(3,5),(3,6),(3,4),(1,5),(1,6),(1,4),(2,5),(2,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{20} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,4),(2,5),(1,6),(2,4),(3,5),(2,6),(3,4),(1,5),(3,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{21} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,4),(3,5),(1,6),(2,4),(1,5),(2,6),(3,4),(2,5),(3,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{22} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,4),(3,5),(1,6),(2,4),(1,5),(2,6),(3,4),(2,5),(3,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{23} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,4),(1,5),(2,6),(3,4),(2,5),(3,6),(1,4),(3,5),(1,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{24} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,4),(3,5),(2,6),(3,4),(1,5),(3,6),(1,4),(2,5),(1,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{25} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,4),(1,5),(3,6),(1,4),(2,5),(1,6),(2,4),(3,5),(2,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{26} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,4),(2,5),(3,6),(1,4),(3,5),(1,6),(2,4),(1,5),(2,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{27} d_1 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,4),(1,5),(1,6),(3,4),(2,5),(2,6),(1,4),(3,5),(3,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_1 d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,4),(1,5),(1,6),(1,4),(2,5),(2,6),(2,4),(3,5),(3,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_2 d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,5),(1,6),(1,4),(2,5),(2,6),(2,4),(3,5),(3,6),(3,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_3 d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,5),(2,6),(2,4),(3,5),(3,6),(3,4),(1,5),(1,6),(1,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_4 d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,5),(3,6),(3,4),(1,5),(1,6),(1,4),(2,5),(2,6),(2,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_5 d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,5),(2,6),(3,4),(2,5),(3,6),(1,4),(3,5),(1,6),(2,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_6 d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,5),(3,6),(2,4),(2,5),(1,6),(3,4),(3,5),(2,6),(1,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_7 d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,5),(1,6),(3,4),(3,5),(2,6),(1,4),(1,5),(3,6),(2,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_8 d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,5),(3,6),(1,4),(3,5),(1,6),(2,4),(1,5),(2,6),(3,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_9 d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,5),(2,6),(1,4),(1,5),(3,6),(2,4),(2,5),(1,6),(3,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{10} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,5),(1,6),(2,4),(1,5),(2,6),(3,4),(2,5),(3,6),(1,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{11} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6),(1,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{12} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,5),(1,6),(3,4),(2,5),(2,6),(1,4),(3,5),(3,6),(2,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{13} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,5),(2,6),(1,4),(3,5),(3,6),(2,4),(1,5),(1,6),(3,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{14} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,5),(2,6),(3,4),(3,5),(3,6),(1,4),(1,5),(1,6),(2,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{15} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,5),(3,6),(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{16} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,5),(3,6),(2,4),(1,5),(1,6),(3,4),(2,5),(2,6),(1,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{17} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,5),(2,6),(2,4),(2,5),(3,6),(3,4),(3,5),(1,6),(1,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{18} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,5),(2,6),(2,4),(1,5),(3,6),(3,4),(2,5),(1,6),(1,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{19} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,5),(3,6),(3,4),(2,5),(1,6),(1,4),(3,5),(2,6),(2,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{20} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,5),(3,6),(3,4),(3,5),(1,6),(1,4),(1,5),(2,6),(2,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{21} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,5),(2,6),(1,4),(2,5),(3,6),(2,4),(3,5),(1,6),(3,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{22} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,5),(3,6),(1,4),(2,5),(1,6),(2,4),(3,5),(2,6),(3,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{23} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,5),(1,6),(2,4),(3,5),(2,6),(3,4),(1,5),(3,6),(1,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{24} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,5),(3,6),(2,4),(3,5),(1,6),(3,4),(1,5),(2,6),(1,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{25} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,5),(2,6),(3,4),(1,5),(2,6),(1,4),(2,5),(3,6),(2,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{26} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,5),(2,6),(3,4),(1,5),(3,6),(1,4),(2,5),(1,6),(2,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{27} d_2 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,5),(1,6),(1,4),(3,5),(2,6),(2,4),(1,5),(3,6),(3,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{1} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,5),(1,6),(1,4),(1,5),(2,6),(2,4),(2,5),(3,6),(3,4) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{2} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,6),(1,4),(1,5),(2,6),(2,4),(2,5),(3,6),(3,4),(3,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{3} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,6),(2,4),(2,5),(3,6),(3,4),(3,5),(1,6),(1,4),(1,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{4} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,6),(3,4),(3,5),(1,6),(1,4),(1,5),(2,6),(2,4),(2,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_5 d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,6),(2,4),(3,5),(2,6),(3,4),(1,5),(3,6),(1,4),(2,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_6 d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,6),(3,4),(2,5),(2,6),(1,4),(3,5),(3,6),(2,4),(1,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_7 d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,6),(1,4),(3,5),(3,6),(2,4),(1,5),(1,6),(3,4),(2,6) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_8 d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,6),(3,4),(1,5),(3,6),(1,4),(2,5),(1,6),(2,4),(3,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_9 d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,6),(2,4),(1,5),(1,6),(3,4),(2,5),(2,6),(1,4),(3,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{10} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,6),(1,4),(2,5),(1,6),(2,4),(3,5),(2,6),(3,4),(1,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{11} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,6),(1,4),(2,5),(2,6),(2,4),(3,5),(3,6),(3,4),(1,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{12} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,6),(1,4),(3,5),(2,6),(2,4),(1,5),(3,6),(3,4),(2,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{13} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,6),(2,4),(1,5),(3,6),(3,4),(2,5),(1,6),(1,4),(3,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{14} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,6),(2,4),(3,5),(3,6),(3,4),(1,5),(1,6),(1,4),(2,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{15} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,6),(3,4),(1,5),(1,6),(1,4),(2,5),(2,6),(2,4),(3,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{16} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,6),(3,4),(2,5),(1,6),(1,4),(3,5),(2,6),(2,4),(1,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{17} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6),(1,4),(1,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{18} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,6),(2,4),(2,5),(1,6),(3,4),(3,5),(2,6),(1,4),(1,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{19} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,6),(3,4),(3,5),(2,6),(1,4),(1,5),(3,6),(2,4),(2,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{20} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,6),(3,4),(3,5),(3,6),(1,4),(1,5),(1,6),(2,4),(2,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{21} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,6),(2,4),(1,5),(2,6),(3,4),(2,5),(3,6),(1,4),(3,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1) f_{22} d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (1,6),(3,4),(1,5),(2,6),(1,4),(2,5),(3,6),(2,4),(3,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1)f_{23}d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,6),(1,4),(2,5),(3,6),(2,4),(3,5),(1,6),(3,4),(1,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1)f_{24}d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,6),(3,4),(2,5),(3,6),(1,4),(3,5),(1,6),(2,4),(1,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1)f_{25}d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,6),(1,4),(3,5),(1,6),(2,4),(1,5),(2,6),(3,4),(2,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1)f_{26}d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (3,6),(2,4),(3,5),(1,6),(3,4),(1,5),(2,6),(1,4),(2,5) \end{pmatrix}$$

$$(\Gamma_1 \times \Delta_1)f_{27}d_3 = \begin{pmatrix} (1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6) \\ (2,6),(1,4),(1,5),(3,6),(2,4),(2,5),(1,6),(3,4),(3,5) \end{pmatrix}$$

Renaming the symbols we have:

$(1,4) \to 1$, $(1,5) \to 2$, $(1,6) \to 3$, $(2,4) \to 4$, $(2,5) \to 5$, $(2,6) \to 6$, $(3,4) \to 7$, $(3,5) \to 8$, $(3,6) \to 9$

The permutation group in cyclic form is obtained as:

G_1 = {(1), (1 4 7), (1 7 4), (2 5 8), (2 8 5), (3 6 9), (3 9 6), (1 2 3 4 5 6 7 8 9),

(1 2 6 4 5 9 7 8 3), (1 2 6 7 8 3 4 5 9), (1 2 9 7 8 6 4 5 3), (1 2 9 4 5 3 7 8 6),

(1 2 3 7 8 9 4 5 6), (1 3 2 4 6 5 7 9 8), (1 3 2 7 9 8 4 6 5), (1 3 5 4 6 8 7 9 2),

(1 3 5 7 9 2 4 6 8), (1 3 8 4 6 2 7 9 5), (1 3 8 7 9 5 4 6 2), (1 5 3 4 8 6 7 2 9),

(1 5 3 7 2 9 4 8 6), (1 5 6 7 2 3 4 8 9), (1 5 9 4 8 3 7 2 6), (1 5 9 7 2 6 4 8 3),

(1 5 6 4 8 9 7 2 3), (1 6 2 4 9 5 7 3 8), (1 6 2 7 3 8 4 9 5), (1 6 8 4 9 2 7 3 5),

(1 6 8 7 3 5 4 9 2), (1 6 5 8 3 2 4 9 8), (1 6 5 4 9 8 7 3 2), (1 8 3 4 2 6 7 5 9),

(1 8 3 7 5 9 4 2 6), (1 8 6 7 5 3 4 2 9), (1 8 6 4 2 9 7 5 3), (1 8 9 4 2 3 7 5 6),

(1 8 9 7 5 6 4 2 3), (1 9 2 7 6 8 4 3 5), (1 9 2 4 3 5 7 6 8), (1 9 5 4 3 8 7 6 2),

(1 9 5 7 6 2 4 3 8), (1 9 8 4 3 2 7 6 5), (1 9 8 7 6 5 4 3 2), (1 4 7)(3 9 8), (1 4 7)(2 8 5),

(1 4 7)(3 6 9), (1 4 7)(2 5 8), (1 7 4)(2 8 5), (1 7 4)(3 6 9), (1 7 4)(3 9 6), (1 7 4)(2 5 8),

(2 8 5)(3 6 9), (2 8 5)(3 9 6), (2 5 8)(3 6 9), (2 5 8)(3 9 6), (1 2 6)(3 7 8)(4 5 9),

(1 2 9)(3 4 5)(6 7 8), (1 2 3)(4 5 6)(7 8 9), (1 4 7)(2 8 5)(3 9 6), (1 4 7)(2 8 5)(3 6 9),

(1 4 7)(2 5 8)(3 9 6), (1 4 7)(2 5 8)(3 6 9), (1 5 9)(2 6 7)(3 4 8), (1 5 6)(2 3 7)(4 8 9),

(1 5 3)(2 9 7)(4 8 6), (1 6 8)(2 4 9)(3 5 7), (1 6 2)(3 8 7)(5 4 9), (1 6 5)(2 7 3)(4 9 8),

(1 7 4)(2 8 5)(3 9 6), (1 7 4)(2 8 5)(3 6 9), (1 7 4)(2 5 8)(3 9 6), (1 7 4)(2 5 8)(3 6 9),

(1 8 6)(2 9 4)(3 7 5), (1 8 3)(2 6 4)(2 5 7), (1 8 9)(2 3 4)(5 6 7), (1 9 5)(2 7 6)(3 8 4),

(1 9 8)(2 4 3)(5 7 6), (1 9 2)(3 5 4)(6 8 7), (1 3 2)(4 6 5)(7 9 8), (1 3 5)(2 7 9)(4 6 8),

(1 3 8)(2 4 6)(5 7 9)}

G_1 is a p-group of order 81 given by $|G_1| = 81 = 3^4$

3.3.2 Consider the permutation groups A_2 and B_2

$A_2 = \{(1),(12345),(13524),(14253),(15432)\}$

$B_2 = \{(1),(67)\}$ *acting on the sets* $\Gamma_2 = \{1,2,3,4,5\}$ *and* $\Delta_2 = \{6,7\}$ *respectively.*

Let $P = A_2^{\Delta_1} = \{f : \Delta_2 \to A_2\}$. *Then* $|P| = |A_2|^{|\Delta_2|} = 5^2 = 25$

The mappings are :

$f_1 : 6 \to (1),\ 7 \to (1)$

$f_2 : 6 \to (12345),\ 7 \to (12345)$

$f_3 : 6 \to (13524),\ 7 \to (13524)$

$f_4 : 6 \to (14253),\ 7 \to (14253)$

$f_5 : 6 \to (15432),\ 7 \to (15432)$

$f_6 : 6 \to (1),\ 7 \to (12345)$

$f_7 : 6 \to (1),\ 7 \to (13524)$

$f_8 : 6 \to (1),\ 7 \to (14253)$

$f_9 : 6 \to (1),\ 7 \to (15432)$

$f_{10} : 6 \to (12345),\ 7 \to (1)$

$f_{11} : 6 \to (12345),\ 7 \to (13524)$

$f_{12} : 6 \to (12345),\ 7 \to (14253)$

$f_{13} : 6 \to (12345),\ 7 \to (15432)$

$f_{14} : 6 \to (13524),\ 7 \to (1)$

$f_{15} : 6 \to (13524),\ 7 \to (12345)$

$f_{16} : 6 \to (13524),\ 7 \to (14253)$

$f_{17} : 6 \to (13524),\ 7 \to (15432)$

$f_{18} : 6 \to (14253),\ 7 \to (1)$

$f_{19} : 6 \to (14253),\ 7 \to (12345)$

$f_{20} : 6 \to (14253),\ 7 \to (13524)$

$f_{21} : 6 \to (14253),\ 7 \to (15432)$

$f_{22} : 6 \to (15432),\ 7 \to (1)$

$f_{23} : 6 \to (15432),\ 7 \to (12345)$

$f_{24} : 6 \to (15432),\ 7 \to (13524)$

$f_{25} : 6 \to (15432),\ 7 \to (14253)$

It can be verified that P is a group with respect to the operation:

$(f_1 . f_2)(\delta) = f_1(\delta) f_2(\delta)$, *where* $\delta \in \Delta_2$.

We recall the definition of the action of B_2 *on* P *as* $f^d(\delta_1) = f(\delta_1 d^{-1})$
Where $f \in P$, $d \in B_2$ *and* $\delta_1 \in \Delta_2$, *then* B_2 *acts on* P *as a group* (α_1, δ_1)
and (α_2, δ_2) *in* $\Gamma_2 \times \Delta_2$.

We also recall the definition W = A_2 wr B_2, the semi-direct product of P by B_2 in that order, that is, $W = \{(f, d) | f \in P, \delta_1 \in \Delta_2\}$. Now W is a group with respect to the operation $(f_1 d_1)(f_2 d_2) = (f_1 f_2^{d_1^{-1}})(d_1 d_2)$, *and*

accordingly, $d_1 = (1)$, $d_2 = (6,7)$. *Then the elements of* W *are*:

$(f_1 d_1), (f_2 d_1), (f_3 d_1), (f_4 d_1), (f_5 d_1), (f_6 d_1), (f_7 d_1), (f_8 d_1), (f_9 d_1), (f_{10} d_1),$
$(f_{11} d_1), (f_{12} d_1), (f_{13} d_1), (f_{14} d_1), (f_{15} d_1), (f_{16} d_1), (f_{17} d_1), (f_{18} d_1), (f_{19} d_1), (f_{20} d_1),$
$(f_{21} d_1), (f_{22} d_1), (f_{23} d_1), (f_{24} d_1), (f_{25} d_1), (f_1 d_2), (f_2 d_2), (f_3 d_2), (f_4 d_2), (f_5 d_2),$
$(f_6 d_2), (f_7 d_2), (f_8 d_2), (f_9 d_2), (f_{10} d_2), (f_{11} d_2), (f_{12} d_2), (f_{13} d_2), (f_{14} d_2), (f_{15} d_2),$
$(f_{16} d_2), (f_{17} d_2), (f_{18} d_2), (f_{19} d_2), (f_{20} d_2), (f_{21} d_2), (f_{22} d_2), (f_{23} d_2), (f_{24} d_2), (f_{25} d_2)$

Further, $\Gamma_2 \times \Delta_2 = (\beta, \delta) = \{(1,6), (1,7), (2,6), (2,7), (3,6), (3,7), (4,6), (4,7), (5,6), (5,7)\}$
where $\Gamma_2 = \{1,2,3,4,5\}$ *and* $\Delta = \{6,7\}$ $\forall \beta \in \Gamma_2, \delta \in \Delta_2$

Now define the action of W on $\Gamma_2 \times \Delta_2$ as

$(\beta, \delta) fd = (\beta f(\delta), d\delta)$ *where* $\beta \in \Gamma_2$ *and* $\delta \in \Delta_2$.

We obtain the following by the action of W on $\Gamma_2 \times \Delta_2$

$$(1,6)f_1d_1 = (1f_1(6), d_1) = (1(1),6(1)) = (1,6)$$
$$(1,7)f_1d_1 = (1f_1(7), d_1) = (1(1),7(1)) = (1,7)$$
$$(2,6)f_1d_1 = (2f_1(6), d_1) = (2(1),6(1)) = (2,6)$$
$$(2,7)f_1d_1 = (2f_1(7), d_1) = (2(1),7(1)) = (2,7)$$
$$(3,6)f_1d_1 = (3f_1(6), d_1) = (3(1),6(1)) = (3,6)$$
$$(3,7)f_1d_1 = (3f_1(7), d_1) = (3(1),7(1)) = (3,7)$$

$$(4,6)f_1d_1 = (4f_1(6), d_1) = (4(1),6(1)) = (4,6)$$
$$(4,7)f_1d_1 = (4f_1(7), d_1) = (4(1),7(1)) = (4,7)$$
$$(5,6)f_1d_1 = (5f_1(6), d_1) = (5(1),6(1)) = (5,6)$$
$$(5,7)f_1d_1 = (5f_1(7), d_1) = (5(1),7(1)) = (5,7)$$

and in summary, we form the permutations as below:

$$(\Gamma_2 \times \Delta_2)f_1d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2)f_2d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7),(1,6),(1,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2)f_3d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (3,6),(3,7),(4,6),(4,7),(5,6),(5,7),(1,6),(1,7),(2,6),(2,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2)f_4d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (4,6),(4,7),(5,6),(5,7),(1,6),(1,7),(2,6),(2,7),(3,6),(3,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2)f_5d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (5,6),(5,7),(1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2)f_6d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (1,6),(2,7),(2,6),(3,7),(3,6),(4,7),(4,6),(5,7),(5,6),(1,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2)f_7d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (1,6),(3,7),(2,6),(4,7),(3,6),(5,7),(4,6),(1,7),(5,6),(2,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2)f_8d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (1,6),(4,7),(2,6),(5,7),(3,6),(1,7),(4,6),(2,7),(5,6),(3,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2)f_9d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (1,6),(5,7),(2,6),(1,7),(3,6),(2,7),(4,6),(3,7),(5,6),(4,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2)f_{10}d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (2,6),(1,7),(3,6),(2,7),(4,6),(3,7),(5,6),(4,7),(1,6),(5,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{11} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (2,6),(3,7),(3,6),(4,7),(4,6),(5,7),(5,6),(1,7),(1,6),(2,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{12} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (2,6),(4,7),(3,6),(5,7),(4,6),(1,7),(5,6),(2,7),(1,6),(3,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{13} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (2,6),(5,7),(3,6),(1,7),(4,6),(2,7),(5,6),(3,7),(1,6),(4,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{14} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (3,6),(1,7),(4,6),(2,7),(5,6),(3,7),(1,6),(4,7),(2,6),(5,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{15} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (3,6),(2,7),(4,6),(3,7),(5,6),(4,7),(1,6),(5,7),(2,6),(1,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{16} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (3,6),(4,7),(4,6),(5,7),(5,6),(1,7),(1,6),(2,7),(2,6),(3,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{17} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (3,6),(5,7),(4,6),(1,7),(5,6),(2,7),(1,6),(3,7),(2,6),(4,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{18} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (4,6),(1,7),(5,6),(2,7),(1,6),(3,7),(2,6),(4,7),(3,6),(5,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{19} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (4,6),(2,7),(5,6),(3,7),(1,6),(4,7),(2,6),(5,7),(3,6),(1,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{20} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (4,6),(3,7),(5,6),(4,7),(1,6),(5,7),(2,6),(1,7),(3,6),(2,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{21} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (4,6),(5,7),(5,6),(1,7),(1,6),(2,7),(2,6),(3,7),(3,6),(4,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{22} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (5,6),(1,7),(1,6),(2,7),(2,6),(3,7),(3,6),(4,7),(4,6),(5,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{23} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (5,6),(2,7),(1,6),(3,7),(2,6),(4,7),(3,6),(5,7),(4,6),(1,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{24} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (5,6),(3,7),(1,6),(4,7),(2,6),(5,7),(3,6),(1,7),(4,6),(2,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{25} d_1 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (5,6),(4,7),(1,6),(5,7),(2,6),(1,7),(3,6),(2,7),(4,6),(3,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_1 d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (1,7),(1,6),(2,7),(2,6),(3,7),(3,6),(4,7),(4,6),(5,7),(5,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_2 d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (2,7),(2,6),(3,7),(3,6),(4,7),(4,6),(5,7),(5,6),(1,7),(1,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_3 d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (3,7),(3,6),(4,7),(4,6),(5,7),(5,6),(1,7),(1,6),(2,7),(2,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_4 d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (4,7),(4,6),(5,7),(5,6),(1,7),(1,6),(2,7),(2,6),(3,7),(3,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_5 d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (5,7),(5,6),(1,7),(1,6),(2,7),(2,6),(3,7),(3,6),(4,7),(4,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_6 d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_7 d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (1,7),(3,6),(2,7),(4,6),(3,7),(5,6),(4,7),(1,6),(5,7),(2,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_8 d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (1,7),(4,6),(2,7),(5,6),(3,7),(1,6),(4,7),(2,6),(5,7),(3,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_9 d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (1,7),(5,6),(2,7),(1,6),(3,7),(2,6),(4,7),(3,6),(5,7),(4,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{10} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (2,7),(1,6),(3,7),(2,6),(4,7),(3,6),(5,7),(4,6),(1,7),(5,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{11} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7),(1,6),(1,7),(2,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{12} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (2,7),(4,6),(3,7),(5,6),(4,7),(1,6),(5,7),(2,6),(1,7),(3,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{13} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (2,7),(5,6),(3,7),(1,6),(4,7),(2,6),(5,7),(2,6),(1,7),(4,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{14} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (3,7),(1,6),(4,7),(2,6),(5,7),(3,6),(1,7),(4,6),(2,7),(5,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{15} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (3,7),(2,6),(4,7),(3,6),(5,7),(4,6),(1,7),(5,6),(2,7),(1,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{16} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (3,7),(4,6),(4,7),(5,6),(5,7),(1,6),(1,7),(2,6),(2,7),(3,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{17} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (3,7),(5,6),(4,7),(1,6),(5,7),(2,6),(1,7),(3,6),(2,7),(4,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{18} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (4,7),(1,6),(5,7),(2,6),(1,7),(3,6),(2,7),(4,6),(3,7),(5,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{19} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (4,7),(2,6),(5,7),(3,6),(1,7),(4,6),(2,7),(5,6),(3,7),(1,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{20} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (4,7),(3,6),(5,7),(4,6),(1,7),(5,6),(2,7),(1,6),(3,7),(2,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{21} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (4,7),(5,6),(5,7),(1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{22} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (5,7),(1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{23} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (5,7),(2,6),(1,7),(3,6),(2,7),(4,6),(3,7),(5,6),(4,7),(1,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{24} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (5,7),(3,6),(1,7),(4,6),(2,7),(5,6),(3,7),(1,6),(4,7),(2,6) \end{pmatrix}$$

$$(\Gamma_2 \times \Delta_2) f_{25} d_2 = \begin{pmatrix} (1,6),(1,7),(2,6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7) \\ (5,7),(4,6),(1,7),(5,6),(2,7),(1,6),(3,7),(2,6),(4,7),(3,6) \end{pmatrix}$$

Renaming the symbols we have:

$(1,6) \to 1, (1,7) \to 2, (2,6) \to 3, (2,7) \to 4, (3,6) \to 5, (3,7) \to 6, (4,6) \to 7,$
$(4,7) \to 8, (5,6) \to 9, (5,7) \to 10$

The permutations in cyclic form are:

G_2= {(1), (1 3 5 7 9), (1 5 9 3 7), (1 7 3 9 5), (1 9 7 5 3), (2 10 8 6 4),

(2 4 6 8 10), (2 6 10 4 8), (2 8 4 10 6), (1 2 3 4 5 6 7 8 9 10), (1 2 5 6 9 10 3 4 7 8),

(1 2 7 8 3 4 9 10 5 6), (1 2 9 10 7 8 5 6 3 4), (1 4 3 6 5 8 7 10 9 2),

(1 4 5 8 9 2 3 6 7 10), (1 4 7 10 3 6 9 2 5 8), (1 4 9 27 10 5 8 3 6),

(1 6 3 8 5 10 7 2 9 4), (1 6 5 10 9 4 3 8 7 2), (1 6 7 2 3 8 9 4 5 10),

(1 6 9 47 2 5 10 3 8), (1 8 3 10 5 2 7 4 9 6), (1 8 5 2 9 6 3 10 7 4),

(1 8 7 4 3 10 9 6 5 2), (1 8 9 67 4 5 2 3 10), (1 10 3 2 5 4 7 6 9 8),

(1 10 5 4 9 8 3 2 7 6), (1 10 7 6 3 2 9 8 4 5 4), (1 10 9 8 7 6 5 4 3 2),

(1 3 5 7 9)(2 10 8 6 4), (1 3 5 7 9)(2 4 6 8 10), (1 3 5 7 9)(2 8 4 10 6),

(1 3 5 7 9)(2 6 10 4 8), (1 5 9 3 7)(2 6 10 4 8), (1 5 9 3 7)(2 8 4 10 6),

(1 5 9 3 7)(2 4 6 810), (1 5 9 3 7)(2 10 8 6 4), (1 7 3 9 5)(2 4 6 8 10),

(1 7 3 9 5)(2 6 10 4 8), (1 7 3 9 5)(2 10 8 6 4), (1 7 3 9 5)(2 8 4 10 6),

(1 9 7 5 3)(2 10 7 6 4), (1 9 7 5 3)(2 4 6 8 10), (1 9 7 53)(2 6 10 4 8),

(1 9 7 5 3)(2 8 4 10 6), (1 2)(3 4)(5 6)(7 6)(9 10), (1 5)(2 9)(3 6)(5 8)(7 10),

(1 6)(2 7)(3 8)(4 9)(5 10), (1 8)(2 5)(3 10)(4 7)(6 9), (1 10)(2 3)(4 5)(6 7)(8 9)}

Now, $|G_2| = 50 = 2 \times 5^2$

G_2 has a unique Sylow 5-Subgroup H_2 of order 25 given by

H_2 = {(1), (1 3 5 7 9), (1 5 9 3 7), (1 7 3 9 5), (1 9 7 5 3), (2 10 8 6 4), (2 4 6 8 10),

(2 6 10 4 8), (2 8 4 10 6), (1 3 5 7 9)(2 10 8 6 4), (1 3 5 7 9)(2 4 6 8 10),

(1 3 5 7 9)(2 8 4 10 6), (1 3 5 7 9)(2 6 10 4 8), (1 5 9 3 7)(2 6 10 4 8),

(1 5 9 3 7)(2 8 4 10 6), (1 5 9 3 7)(2 4 6 8 10), (1 5 9 3 7)(2 10 8 6 4),

(1 7 3 9 5)(2 4 6 8 10), (1 7 3 9 5)(2 6 10 4 8),

(1 7 3 9 5)(2 10 8 6 4), (1 7 3 9 5)(2 8 4 10 6), (1 9 7 5 3)(2 10 7 6 4),

(1 9 7 5 3)(2 4 6 8 10), (1 9 7 5 3)(2 6 10 4 8), (1 9 7 5 3)(2 8 4 10 6)}

Thus, $|H_2| = 5^2$ which is a p-group

3.3.3 Consider the permutation groups A_3 and B_3

A_3 = {(1), (1 2 3 4 5 6 7), (1 3 5 7 2 4 6), (1 4 7 3 6 2 5), (1 5 2 6 3 7 4),

(1 6 4 2 7 5 3), (1765432)}

B_3 = {(1),(8 9)} acting on the sets Γ_3 = {1,2,3,4,5} and Δ_3 = {8,9} respectively.

Let $P = A_3^{\Delta_3} = \{f : \Delta_3 \to A_3\}$. *Then* $|P| = |A_3|^{|\Delta_3|} = 7^2 = 49$

After following the same procedure as in 3.3.1, we obtain the permutations in cyclic form as:

G_3 = {(1), (1 3 5 7 9 11 13), (2 4 6 8 10 12 14), (1 5 9 13 3 7 11), (2 6 10 14 4 8 12),

(1 7 13 5 11 3 9), (1 8 14 6 12 4 10), (1 9 3 11 5 13 7), (2 10 4 12 6 14 8),

(1 11 7 3 13 9 5), (2 12 8 4 14 10 6), (1 13 11 9 7 5 3), (2 14 12 10 8 6 4),

(2 4 6 8 10 12 14), (2 6 10 14 4 8 12), (2 8 14 6 12 4 10),

(2 12 8 4 14 10 6), (2 10 4 12 6 14 8), (2 14 12 10 8 6 4), (1 3 5 7 9 11 13),

(1 3 5 7 9 11 13)(2 6 10 14 4 8 12), (1 3 5 7 9 11 13)(2 8 14 6 2 4 10),

(1 3 5 7 9 11 13), (2 10 4 12 6 14 8), (1 3 5 7 9 11 13), (2 12 8 4 14 10 6),

(1 3 5 7 9 11 13)(2 14 12 10 8 6 4), (1 5 9 13 3 7 11), (1 5 9 13 3 7 11)

(2 4 6 8 10 12 14), (1 5 9 13 3 7 11)(2 8 14 6 12 4 10),

(1 5 9 13 3 7 11)(2 10 4 12 6 14 8), (1 5 9 13 3 7 11)(2 12 8 4 14 10 6),

(1 5 9 13 3 7 11)(2 14 12 10 8 6 4), (1 7 13 5 11 3 9),

(1 7 13 5 11 3 9)(2 4 6 8 10 12 14), (1 7 13 5 11 3 9)(2 6 10 14 4 8 12),

(1 7 13 5 11 3 9)(2 10 4 12 6 14 8), (1 7 13 5 11 3 9)(2 12 8 4 14 10 6),

(1 7 13 5 11 3 9)(2 14 12 10 8 6 4), (1 9 3 11 5 13 7),

(1 9 3 11 5 13 7)(2 4 6 8 10 12 14), (1 9 3 11 5 13 7)(2 6 10 14 4 8 12),

(1 9 3 11 5 13 7)(2 8 14 6 12 4 10), (1 9 3 11 5 13 7)(2 12 8 4 14 10 6),

(1 9 3 11 5 13 7)(2 14 12 10 8 6 4), (1 11 7 3 13 9 5),

(1 11 7 3 13 9 5)(2 4 6 8 10 12 14), (1 11 7 3 13 9 5)(2 6 10 14 4 8 12),

(1 11 7 3 1 3 9 5)(2 8 14 6 12 4 10), (1 11 7 3 13 9 5),(2 10 4 12 6 14 8),

(1 11 7 3 13 9 5)(2 14 12 10 8 6 4), (1 13 11 9 7 5 3),

(1 13 11 9 7 5 3)(2 4 6 8 10 12 14), (1 13 11 9 7 5 3)(2 6 10 14 4 8 12),

(1 13 11 9 7 5 3)(2 8 14 6 12 4 10), (1 13 11 9 7 5 3)(2 10 4 12 6 1 4 8),

(1 13 11 9 7 5 3)(2 12 8 4 14 10 6), (1 2)(3 4)(5 6)(7 8)(9 10)(11 12)(1 3 14),

(1 4 5 8 9 12 13 2 3 6 7 10 11 14), (1 6 9 14 3 8 11 2 5 10 13 4 7 12),

(1 8 13 6 11 4 9 2 7 14 5 12 3 10), (1 10 3 12 5 4 7 2 9 4 11 6 13 8),

(1 12 7 4 13 10 5 2 11 8 3 14 9 6), (1 14 11 10 7 6 3 2 13 12 9 8 5 4),

(1 2 3 4 5 6 7 8 9 10 11 12 13 14), (1 2 5 6 9 10 13 14 3 4 7 8 11 12),

(1 2 7 8 13 14 5 6 11 12 3 4 9 10),

(1 2 9 10 3 4 11 1 2 5 6 13 14 7 8), (1 2 11 12 7 8 3 4 13 14 9 10 5 6),

(1 2 13 14 11 12 9 10 7 8 5 6 3 4), (1 4 3 6 5 8 7 10 9 12 11 14 13 2),

(1 4 7 9 13 2 5 8 11 14 3 6 9 12), (1 4 9 12 3 6 11 14 5 8 13 2 7 10),

(1 4 11 14 7 10 3 6 13 2 9 12 5 8), (1 4 13 2 11 14 9 12 7 10 5 8 3 6),

(1 4)(2 13)(3 6)(5 8)(7 10)(9 12)(11 14), (1 6 5 10 9 14 13 4 3 8 7 12 11 2),

(1 6 7 12 13 4 5 10 11 2 3 8 9 14), (1 6 11 2 7 12 3 8 13 4 9 14 5 10),

(1 6 13 4 11 2 9 14 7 12 5 10 3 8), (1 6)(2 11)(3 8)(4 13)(5 10)(7 12)(9 14),

(1 6 3 8 5 10 7 12 9 14 11 2 13 4), (1 8 7 14 13 6 5 12 11 4 3 10 9 2),

(1 8 9 2 3 10 11 4 5 12 13 6 8 1 4), (1 8 11 4 7 14 3 10 13 6 9 2 5 12),

(1 8)(2 9)(3 10)(4 11)(5 12)(6 13)(7 14), (1 8 13 10 5 12 7 14 9 2 11 4 13 6),

(1 8 5 12 9 2 13 6 3 10 7 14 11 4), (1 10 9 4 3 12 11 6 5 14 13 8 7 2),

(1 10 11 6 7 2 3 12 13 8 9 4 5 14), 1 10 13 8 11 6 9 4 7 2 5 14 3 12),

(1 10)(2 7)(3 12)(4 9)(5 14)(6 11)(8 13), (1 10 5 14 9 4 13 8 3 12 7 2 11 6),

(1 10 7 2 13 8 5 14 11 6 3 12 9 4), (1 12 11 8 7 4 3 14 13 10 9 6 5 2),

(1 12 13 14 5 2 7 4 9 6 7 4 5 2 3 14), (1 12)(2 5)(3 14)(4 7)(6 9)(8 11)(10 13),

(1 12 3 14 5 2 7 4 9 6 11 8 13 10), (1 12 5 2 9 6 13 10 3 14 7 4 11 8),

(1 12 9 6 3 14 11 8 5 2 13 10 7 4), (1 14 13 12 11 10 9 8 7 6 5 4 3 2),

(1 14)(2 3)(4 5)(6 7)(8 9)(10 11)(12 13), (1 14 3 2 5 4 7 6 9 8 11 10 13 12),

(1 14 5 4 9 8 13 12 3 2 7 6 11 10), (1 14 7 6 13 12 5 4 11 10 3 2 9 8),

(1 14 9 8 3 2 11 10 5 4 13 12 7 6)}

Now, $|G_3| = 98 = 2 \times 7^2$

G_3 has a unique Sylow 7-Subgroup H_3 of order 49 given by:

H_3 = {(1), (1 3 5 7 9 11 13)(2 4 6 8 10 12 14), (1 5 9 13 3 7 11)(2 6 10 14 4 8 12),

(1 7 13 5 11 3 9), (1 8 14 6 12 4 10), (1 9 3 11 5 13 7)(2 10 4 12 6 14 8),

(1 11 7 3 13 9 5)(2 12 8 4 14 10 6), (1 13 11 9 7 5 3)(2 14 12 10 8 6 4),

(2 4 6 8 10 12 14), (2 6 10 14 4 8 12), (2 8 14 6 12 4 10), (2 12 8 4 14 10 6),

(2 10 4 12 6 14 8), (2 14 12 10 8 6 4), (1 3 5 7 9 11 13),

(1 3 5 7 9 11 13)(2 6 10 14 4 8 12), (1 3 5 7 9 11 13)(2 8 14 6 2 4 10),

(1 3 5 7 9 11 13)(2 10 4 12 6 14 8), (1 3 5 7 9 11 13)(2 12 8 4 14 10 6),

(1 3 5 7 9 11 13)(2 14 12 10 8 6 4), (1 5 9 13 3 7 11), (1 5 9 13 3 7 11)(2 4 6 8 10 12 14),

(1 5 9 13 3 7 11)(2 8 14 6 12 4 10), (1 5 9 13 3 7 11)(2 10 4 12 6 14 8),

(1 5 9 13 3 7 11)(2 12 8 4 14 10 6), (1 5 9 13 3 7 11)(2 14 12 10 8 6 4), (1 7 13 5 11 3 9),

(1 7 13 5 11 3 9)(2 4 6 8 10 12 14), (1 7 13 511 3 9)(2 6 10 14 4 8 12),

(1 7 13 5 11 3 9)(2 10 4 12 6 14 8), (1 7 13 5 11 3 9)(2 12 8 4 14 10 6),

(1 7 13 5 11 3 9)(2 14 12 10 8 6 4), (1 9 3 11 5 13 7), (1 9 3 11 5 13 7)(2 4 6 8 10 12 14),

(1 9 3 11 5 13 7)(2 6 10 14 4 8 12), (1 9 3 11 5 13 7)(2 8 14 6 12 4 10),

(1 9 311 5 13 7)(2 12 8 4 14 10 6), (1 9 3 11 5 13 7)(2 14 12 10 8 6 4), (1 11 7 3 13 9 5),

(1 11 7 3 13 9 5)(2 4 6 8 10 12 14), (1 11 7 3 13 9 5)(2 6 10 14 4 8 12),

(1 11 7 3 1 3 9 5)(2 8 14 6 12 4 10), (1 11 7 3 13 9 5)(2 10 4 12 6 14 8),

(1 11 7 3 13 9 5)(2 14 12 10 8 6 4), (1 13 11 9 7 5 3), (1 13 11 9 7 5 3)(2 4 6 8 10 12 14),

(1 13 11 9 7 5 3)(2 6 10 14 4 8 12), (1 13 11 9 7 5 3)(2 8 14 6 12 4 10),

(1 13 11 9 7 5 3)(2 10 4 12 6 1 4 8), (1 13 11 9 7 5 3)(2 12 8 4 14 10 6),

$|H_3| = 7^2 = 49$ *is a p – group*

3.3.4 Consider the permutation groups A_4 and B_4

A_4 = {(1), (1 2 3 4 5), (1 3 5 2 4), (1 4 2 5 3), (1 5 4 3 2)}

B_4 = {(1), (6 7 8), (6 8 7)} acting on the sets

Γ_4 = {1, 2, 3, 4, 5} and Δ_4 = {6, 7, 8} respectively.

Let $P = A_4^{\Delta_4} = \{f : \Delta_4 \to A_4\}$. *Then* $|P| = |A_4|^{|\Delta_4|} = 5^3 = 125$

Applying the same procedure as in 3.3.1, obtain a permutation group in cyclic form as:

G_4 = {(1), (1 4 7 10 13)(2 5 8 11 14)(3 6 9 12 15), (1 7 13 4 10)(2 8 14 5 11)(3 9 15 6 12)

(1 10 4 13 7)(2 11 5 14 8)(3 12 6 15 9), (1 13 10 7 4)(2 14 11 8 5)(3 15 12 9 6),

(2 5 8 11 14)(3 9 15 6 12), (2 5 8 11 14)(3 12 6 15 9), (2 5 8 11 14)(3 15 12 9 6),

(2 8 14 5 11)(3 6 9 12 15), (2 8 14 5 11)(3 12 6 15 9), (2 8 15 5 11)(3 15 12 9 6),

(2 11 5 14 8)(3 6 9 12 15), (2 11 5 14 8)(3 6 9 12 15), (2 11 5 14 8)(3 9 15 6 12),

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(1 6 2 13 3 14 10 15 11 7 12 8 4 9 5), (1 9 5)(2 13 6)(3 14 10)(4 12 8)(7 15 11),

(1 12 8 4 15 11 7 3 14 10 6 2 13 9 5)}

Now, $|G_4| = 375 = 3 \times 5^3$

G_4 has a unique Sylow 5-Subgroup H_4 of order 125 given by:

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(1 4 7 10 13)(2 14 11 8 5)(3 9 15 6 12), (1 4 7 4 13)(2 14 11 8 5)(3 12 6 15 9),
(1 7 13 4 10)(3 6 9 12 15), (1 7 13 4 10)(3 12 6 15 9), (1 7 13 4 10)(3 15 12 9 6),

(1 7 13 4 10)(2 5 8 11 14), (1 7 13 4 10)(3 15 12 9 6), (1 7 13 4 10)(2 5 8 11 14),
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(1 7 13 4 10)(2 11 5 14 8)(3 15 12 9 6), (1 7 13 4 10)(2 14 11 8 5),
(1 7 13 4 10)(2 14 11 8 5)(3 6 9 12 15), (1 7 13 4 10)(2 14 11 8 5)(3 12 6 15 9),
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(1 10 4 13 7)(2 14 11 8 5), (1 10 4 13 7)(2 14 11 8 5)(3 6 9 12 15),
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(1 13 10 7 4)(2 8 14 5 11), (1 13 10 7 4)(2 8 14 5 11)(3 6 9 12 15),
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(1 7 13 4 10)(2 8 14 5 11)(3 6 9 12 15), (1 7 13 4 10)(2 8 14 15 11)(3 12 6 15 9),
(1 7 13 4 10)(2 8 14 15 11)(3 15 12 9 6), (1 7 13 4 10)(3 9 15 6 12),
(1 7 13 4 10)(2 5 8 11 14)(3 9 15 6 12), (1 7 13 4 10)(2 11 5 14 8)(3 9 15 6 12),
(1 7 13 4 10)(2 14 11 8 5)(3 9 15 6 12),(2 8 14 5 11)(3 9 15 6 12),
(1 4 7 10 13)(2 8 14 5 11)(3 9 15 6 12),(1 10 4 13 7)(2 8 11 14 5 11)(3 9 15 6 12),
(1 13 10 7 4)(2 8 14 5 11)(3 9 15 6 12), (1 10 4 13 7)(2 11 5 14 8),
(1 10 4 13 7)(2 11 5 14 8)(3 6 9 12 15), (1 10 4 13 7)(2 11 5 14 8)(3 9 15 6 12),
(1 10 4 13 7)(2 11 5 14 8)(3 15 12 9 6), (1 10 4 13 7)(3 12 6 15 9),
(1 10 4 13 7)(2 5 8 11 14)(3 12 6 15 9), (1 10 4 13 7)(2 8 14 5 11)(3 12 6 15 9),
(1 10 4 13 7)(2 14 11 8 5)(3 12 6 15 9), (2 11 5 14 8)(3 12 6 15 9),
(1 4 7 10 13)(2 11 5 14 8)(3 12 6 15 9), (1 7 13 4 10)(2 11 5 14 8)(3 12 6 15 9),
(1 13 10 7 4)(2 11 5 14 8)(3 12 6 15 9), (1 13 10 7 4)(2 14 11 8 5),
(1 13 10 7 4)(2 14 11 8 5)(3 6 9 12 15), (1 13 10 7 4)(2 14 11 8 5)(3 9 15 6 12),
(1 13 10 7 4)(2 14 11 8 5)(3 12 6 15 9), (1 13 10 7 4)(3 15 12 9 6),
(1 13 10 7 4)(2 5 8 11 14)(3 15 12 9 6), (1 13 10 7 4)(2 8 14 5 11)(3 15 12 9 6),
(1 13 10 7 4)(2 11 5 14 8)(3 15 12 9 6), (2 14 11 8 5)(3 15 12 9 6),
(1 4 7 10 13)(2 14 11 8 5)(3 15 12 9 6), (1 7 13 4 10)(2 14 11 8 5)(3 15 12 9 6),
(1 10 4 13 7)(2 14 11 8 5)(3 15 12 9 6)}

H_4 is a p-group of order 125 given by $|H_4| = 125 = 5^3$

CHAPTER FOUR

4.0 SUMMARY, CONCLUSION AND RECOMMENDATIONS

4.1 Summary and Conclusion

In summary, the study sought to construct p-groups via wreath products.

It also sought to determine the formation of permutations in cyclic forms as well as outlines the procedure for constructing wreath products of groups.

In conclusion, it is realized that p-groups are notified to be the fundamental tool in understanding the structure of groups and in the classification of finite groups since they arise both as subgroups and quotient groups.

Wreath products allow us to construct new groups since the result of a wreath product is itself a group.

4.2 Recommendation

Further research on this study is recommended in the study and construction of some automata.

It is also recommended to investigate the use of wreath products by composers in the creation of new music.

Intended research could also be carried out on the lamplighter group which is a group constructed via wreath product and happens to be an example of a group of exponential growth.

It is further recommended that future computations of the results of wreath products of p-groups should be done using the GAP (Groups, Algorithms Programming) language to ease the long stress of the manual way of obtaining the results.

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