UNIVERSITY FOR DEVELOPMENT STUDIES, TAMALE

APPLICATION OF LINEAR PROGRAMMING TO MEDIA SELECTION PLANNING: A CASE STUDY OF MOBILE TELECOMMUNICATION NETWORK (MTN)

BY

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DECLARATION

Student

I hereby declare that this thesis is the result of my own original work and that no part of it has been presented for another degree in this University or elsewhere.

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ABSTRACT

Generally this research used Linear Programming techniques to investigate the budgetary allocation of MTN Company for effective media selection planning. The problem was formulated as a linear programming technique by using the available data obtained from the company and other media sources. Mathematical software which embodied the simplex algorithm techniques such as Quantitative Manager for Business version 3.2 and Linear Programming Solver version 5.2.2 were used to solve the resulting Linear Programming model. Sensitive analyses were carried out on some selected parameters of interest to test the robustness or otherwise of the revised model to slight variations in the input parameter values. It was also aimed at determining how redundant a constraint was to the solution of the Linear Programming problem. It was found that out of the three major media categories which give a total of Forty-Five (45) media outlets in the country and a budget of Six Hundred Thousand Ghana cedis (GH¢ 600,000), the optimal target audience exposure was 541,399,986. The optimum media mix which generated the desire objective value for the advertising campaign include; three (3) Television media outlet, seven (7) newspapers within the print media outlet, and twenty (20) FM stations within the radio media outlet respectively. We can conclude that modeling the media selection planning problem using Linear Programming techniques for the MTN Company advertising campaign was useful and it can be applied to any area where allocation of fund to optimize the media mix is a setback.

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DEDICATION

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACRONYMS AND ABBREVIATIONS

Ad	Advertisement
AMPL	A Mathematical Programming Language
BFS	Basic Feasible Solution
Exp	Exposure
FM	Frequency Modulo
GP	Goal Programming
GCG	Graphic Communication Group
GBC	Ghana Broadcasting Corporation
GTV	Ghana Television
GH¢	Ghana Cedis
GAMS	Generalized Algebraic Modeling System
HAMM	High Assay Media Model
ILP	Integer Linear Programming
IBFS	Initial Basic Feasible Solution
LHS	Left Hand Side
LP	Linear Programming
MILP	Mixed Integer Linear Programming
MTN	Mobile Telecommunication Network
MEDIAC	Media Evaluation Using Dynamic and Interactive Applications of Computers
MMRC	Media Monitoring Research Centre
OR	Operation Research

QM	Quantitative Manager
RHS	Right Hand Side

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Marketing companies as well as manufacturers carry out advertisement to inform prospective customers about a product, service or remind them of their existence. The advertiser hopes that his advertising message will make people to buy a product or service who would not otherwise do so. The advertiser believes that his advertising investment will bring a profit or some kind of performance measure to the company. It is not only the aforementioned entities that conduct advertisement but also individuals, administrative offices, institutional bodies, social organization etc. conduct advertisement to some extent to promote a cause or tell the public about a service they offer. In any of these scenarios, the advertiser buy a space or time in the media such as TV, Billboards, Radio, Newspaper as a channel to disseminate the information to the prospective target audiences. They make choices in regards to which category or a combination of media categories to choose to deliver the advertising message through to the target audience.

Media selection planning is part of the integrated advertising planning process begun from 1900 – 1960 (Chandon, 1976). During this period, the media selection planning decision tended to be less complex and of less economic importance. This is due to the fact that media categories as well as media vehicles were limited in number, and also the use of advertising as a major promotional tool was less extensive. Again, within this era, the media selection specialists tended to use subjective models which were based on expert judgment and some simple arithmetic models where the data were based on circulation and cost of media. However, as advertising increased its importance and as a promotional tool, and as more sophisticated media information became available, the media selection planning process became increasingly complex. For instance, each year about 27 billion dollars is spent on the eight (8) major media in the United State (Chandon, 1976). This huge expenditure alone allocated to media selection indicates that media decision making in today's marketing environment are of economic importance to the marketing companies. Thus, media selection planning has attracted a wide variety of models building than any other problem in the marketing domain (Kotler, 1971).

In this research paper, we devote attention to the operation of specific optimization techniques -linear programming model to examine the concept of media selection planning problem.

In the media selection planning process, the challenge is to select from among various media alternatives the "best set". Given that the total budget available is a restraint. Alternatives include not only media, but specific choices within a given medium as well. For a given newspaper, for example, there is the choice of page size, colors and the like. Thus, choices available include all media capable of carrying out an advertisement to achieve the desired results. Each medium is expected to yield a certain number of exposures per Ghana cedi. Clearly, the term exposure will be defined in such a way that exposures among various alternatives can be compared within the media categories.

1.2 Statement of the Problem

selection using a conceptual model is considered a serious challenge in Ghana. that most companies are more inclined to very little or no conceptual approach in dealing with most media campaigns. In some instances, it is considered as an insurmountable challenge in the country (Kwansah-Aidoo, 2003). It is in the light of this that the linear programming conceptual model is adopted to expose the clear cut benefits of the conceptual approach. The reason being that, conceptual models allow for optimization of the media selection planning in such a way that, the effect of each variable present in relation to its cost and effect which is completely absent in heuristic approach adopted over the years in Ghana is addressed. For that matter, the use of linear programming as a conceptual model has become inevitable. Linear programming is a natural format for analyzing the media selection problem. It is applied to problems where there are a large number of ways to allocate scarce resources among competing alternatives in order to attain the best possible value of some stated criterion function, subject to a number of limiting constraints. The media budget is the scarce resource and the various media: Television, Radio, and Print represent alternative means of spending the budget allocation. The main constraints in media selection are the size of the media budget and the minimum and maximum usage of specific media. The best combination is determined by some effectiveness criterion. The criterion function is an attempt to have factors - such as the advertiser's target population, the difference in prestige of the various media, and the different exposure values of the various units in each of these influences the decision of the linear programming model.

1.3 Research Objectives

The general objective of the study is to use linear programming techniques to investigate how to allocate advertising budget effectively across the various media so as to achieve the total target audiences' exposure to a selected company, whilst operating under the policies and the restrictions that would be identified. Specifically, the research intends:

- To come out with a quantitative model that will maximize the total target audiences' exposure for the company while working within its budget.
- To determine the optimum media mix for the company.
- To investigate the performance of the model using real data
- To arrive at definite conclusions about the problem.

1.4 Research Questions

Questions asked to guide the researcher include:

- What is the best way to utilize the advertising budget in line with the media selection planning of the selected company?
- What is the optimal media mix for the company?

1.5 Significance of the Study

A linear programming model provides more realistic results which means, aiding the decision makers to make informed decisions about their choices as to the best media selection. Moreover, the performances of the company in line with their marketing objective(s) that might have been conceived would also be achieved at the end of the campaign program. Additionally, prospective media buyers and managers as well as other stakeholders who have similar problem of allocating advertising budget to media selection would find the results of the study useful to them. In conjunction with the above, media buyers and managers would appreciate the use of optimization techniques in addressing advertising problems.

1.6 Justification of the Study

The media selection planning for the MTN Company in Ghana will enable the company to identify the best media mix towards the purchase of advertising space or time to achieve the intended objective. The output of the model will serve as a useful tool in decision making process in this current competitive business environment in the country. Moreover, the study is intended to assist the company to allocate its advertising budget effectively across the various media group that will in turn generate the maximum target audience exposure for the company.

1.7 Delimitation/Scope of the Study

The research work is focused on the determination of the best media mix that will in turn yield the maximum target audience exposure for the advertisement promotion campaign programme for the MTN Company. The study was conducted on just one of the telecommunication company due to limited fund and time constraint. It was extremely costly and time – consuming to extend the study to cover other telecommunication companies operating in the country. In addition, the problem of disclosure of information by the company was encountered during the data collection process.

1.8 Definition of Terms

For ease of reading and understanding of the study as well as the avoidance of any ambiguity in the mind of the reader, the essential terms used in this study are defined as listed below. The definitions apply to the concepts throughout the study except where they are re-defined in specific contexts.

Media Vehicle: is any possible carrier of advertisement. Each vehicle will yield a certain number of exposures per Ghana cedi.

Target Audience: This represents the prospective customers the advertiser is aiming to reach with the advertising message. They may include some group of people at a certain age, or women at certain levels or even career men and women.

Media Budget: This is an amount of money that is set aside to achieve the objective of a particular promotion or advertisement for a given period of time.

Media Selection: This is the selection of the best media-mix that would yield the effective exposure.

Media Planning: refers to the process of selecting media time and space to disseminate advertising messages in order to accomplish marketing objectives.

Advertising: is the process of presenting a copy through a medium to disseminate information to a desired audience

Algorithm: A step-by-step problem-solving procedure, especially an established, recursive computational procedure for solving a problem in a finite number of steps.

Constraint: is some function of the decision variables that must be less than or equal to, greater than or equal to or equal to some specific value.

Decision Variables: these represent the decisions that must be made in the problem.

Linear Programming (LP): a mathematical programming technique which involves creating and solving optimization problems with linear objective functions and linear constraints.

Mathematical Programming: a field of management science that finds the optimal, or most efficient way of using limited resources to achieve the objectives of an individual or a business.

Model: a representation of the reality that captures the essence of reality

Objective Function: it identifies some function of the decision variables that the decision maker wants to optimize.

Optimal Solution: it is the best answer to the problem given the constraints

Optimization: to make something function at its best or most effective, or use something to its best advantage.

Shadow Price: it identifies the amount by which the objective function value changes given a unit increase in the right hand side value of the constraints, under the assumption that all other distribution costs remain constant.

Simplex Method: an efficient way of solving LP problems

1.9 Organization of the Study

The study is organized in five chapters. Chapter One deals with the background information of the study, statement of the problem, objectives, research questions, operational definition of terms. Chapter Two focuses on review of the relevant literature which covers brief concept of media selection planning, overview of linear programming, and other mathematical models. The methodology used for the study is presented in

Chapter Three. Chapter Four is based on computational results and sensitivity analysis in addition to summary of information. The last chapter which is Chapter Five encompasses conclusion, recommendation, and direction for future work.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

Optimization or constraint optimization is the process of obtaining the best possible 'result' under the 'circumstance'. The result is measured in terms of an objective which is either to maximize or minimize, whilst the circumstances are defined by a set of equality and/or inequality constraints (Murtagh and Saunders, 1978).

Optimization originated from George Dantzig, a member of the United State Air Force who developed the simplex method for solving constraint optimization problems in (1947). The essence of the algorithm is to provide an efficient method in solving programming problems in which the objective and constraints have linear structure. Since then, experts from a variety of fields especially mathematics, Operation Research, Economics etc. have developed the theory behind linear programming and explored its application (Lewis, 2008).

Linear programming which is one of the most widely used techniques in solving optimization problems has found practical application in many facets of human endeavor. Examples are financial portfolios construction, transportation systems, manufacturing, Health etc. These aids others are typical areas where LP analysis have been applied (Bazaraa *et al.*, 2005).

Today, optimization comprises a wide variety of techniques from Operations Research, Artificial Intelligence and Computer Science, and is used to improve business processes.

The word "Programming" in Mathematical Programming is of a different flavor than the "programming" in Computer Programming. In the former case, it means to plan and organize. In the latter case, it means to write instructions for performing calculations. Although aptitude in one suggests aptitude in the other, training in the one kind of programming has very little direct relevance to the other (Konno and Yamazaki, 1991).

2.2 Overview of Linear Programming

LP is concerned with the theory and methods of maximizing or minimizing a linear function subject to linear constraints. In management terms, it could be said to be finding the best or efficient way to utilize scarce resources (Twum, 2011). The most difficult aspect of solving a constrained optimization problem by LP is to formulate or state the problem in a linear programming format or framework (Catherine, 2008). Depending on the number of decision variables, LP problem are easily solved graphically or by the use of computers (Thomas, 1998). It is important, however, to know the process by which even the most complex LP problems are formulated and solved and how the results are interpreted. In this study we determine the optimal media mix in a media selection planning using LP as a tool subject to constraints based on policies, restrictions, budgetary and others of the organization. LP begins with the construction of a mathematical model to represent a real problem. The general LP model is of the form:

Optimize: $f(x_1, x_2, \dots, x_n)$

Subject to: $g_i(x_1, x_2, \dots, x_n) \leq b_i 1 \ \leq i \leq p$

$g_i(x_1, x_2, \dots, x_n) \leq b_i p + 1 \ \leq i \leq k$

$g_i(x_1, x_2, \dots, x_n) \leq b_i k + 1 \ \leq i \leq m$

$$x_1, x_2, \dots, x_n \geq 0$$

The functions $f(x_1, x_2, \dots, x_n)$ and $g_i(x_1, x_2, \dots, x_n), 1 \leq i \leq m$islinear. The decision variables $x_j = (x_1, x_2, \dots, x_n)^T$ values which optimize the objective function of the LP model above and satisfy all the constraints equations and inequalities are referred to as the "optimal decision". The term b_i,$(1 \leq i \leq \text{m})$ is called the right-hand-side (RHS) and represents the upper and lower limit imposed on the ith constraint function. The conditions $x_1, x_2, \dots, x_n \geq 0$ restrict the decision variables to non-negative real numbers only. Since the objective and constraints function are linear they are precisely defined in the form:

$$f(x_1, x_2, \dots, x_n) = c_1x_1 + c_2x_2 + \dots + c_nx_n = \sum_{j=1}^{n} c_jx_j \qquad (2.1)$$

$$g_i(x_1, x_2, \dots, x_n) = a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = \sum_{j=1}^{n} a_{ij}x_j \qquad (2.2)$$

The LP in (2.1) and (2.2) can thus be written in the following form:

Optimize: $\sum_{j=1}^{n} c_j x_j$

Subject to: $\sum_{j=1}^{n} a_{ij}x_j \leq b_i 1 \leq i \leq p$

$\sum_{j=1}^{n} a_{ij}x_j = b_i p + 1 \leq i \leq k$

$\sum_{j=1}^{m} a_{ij}x_j \geq b_i k + 1 \leq i \leq m$

$x_j \geq 0 1 \leq j \leq n$

The quantities a_{ij} and c_j are called Technological and Cost Coefficients respectively. These together with the RHS b_i constitute the main parameters of the models.

2.2.1 The Canonical and Standard form LP

The LP is in Canonical form when given as:

Optimize: $\sum_{j=1}^{n} c_j x_j$

Subject to: $\sum_{j=1}^{n} a_{ij}x_j \leq b_i$, for all i,

$x_j \geq 0$, for all j.

The LP is in Standard form when given as:

Optimize: $\sum_{j=1}^{n} c_j x_j$

Subject to: $\sum_{j=1}^{n} a_{ij}x_j = b_i$, for all i,

$x_j \geq 0$, for all j,

All LP problems can be transformed into the standard form. The standard form LP is very important since LP algorithms work under equality conditions only. The standard and canonical form LPs may be expressed in the matrix form in the following forms respectively:

Optimize: $C^T x$

Subject to: $Ax \leq b$, $x \geq 0$ and

Optimize: $C^T x$

Subject to: $Ax = b$, $x \geq 0$,

Where

$(c_1, c_2, \dots, c_{n},)^T$, $b = (b_1, b_2, \dots, b_{n},)^T$, $x = (x_1, x_2, \dots, x_{n},)^T$ and

$$A = \begin{pmatrix} a_{11} & a_{12}a_{13} \dots \dots a_{1n} \\ a_{21} & a_{22}a_{23} \dots \dots \dots a_{2n} \\ \dots & \dots \\ \dots & \dots \\ \dots & \dots \\ \dots & \dots \\ a_{m1}a_{m2}a_{m3} & \dots \dots \dots a_{mn} \end{pmatrix}$$

2.2.2 Assumptions of Linear Programming

A Linear Programming model has four main inherent assumptions namely; Proportionality, Additivity, Divisibility and Certainty (Shapiro, 1984).

2.2.2.1 Proportionality

This suggests that the contribution of any variable to the objective function or constraints is proportional to that value of the variable. Thus an activity level x_k $(1 \leq k \leq n)$ contributes $c_k x_k$ and $a_{ik} x_k$ respectively to the objective and the constraint functions. The contribution to both is proportional to the activity level. However there may be cases where this assumption may not hold precisely but can be used as an approximation.

2.2.2.2 Additivity

In this case the contribution of any variable to the objective function or constraints is independent of the values of the other variables. It supposes that there are no interactions within or between any of the activity levels so that terms such as x_k^2 or $x_k x_{k+1}$ where $1 \leq k \leq n$ are non-existent in either the objective function or the constraint functions. The assumption implies that given activity levels x_k and x_p, say, their total contribution to both the objective function and constraint function is the sum of their individual contributions.

2.2.2.3 Divisibility

This assumption requires that an activity level may be divided into any fractional levels so that non-integer values are admissible. In cases where integer values are strictly required the values obtained after solving LP model may be rounded to the nearest integer. This practice may not always be useful, since such a modification can seriously result in a sub-optimal solution where the Linear Programming model is sensitive.

2.2.2.4 Certainty

This assumption is also called the deterministic assumption. This means that all parameters (all coefficients in the objective function and the constraints) are known with certainty. Realistically, however, coefficients and parameters are often the result of guess-work and approximation. The effect of changing these numbers can be determined with sensitivity analysis.

2.2.3 The Simplex Method

The Simplex method is a widely used algorithm for solving large scale LP problems in particular where ones geometric intuition falters and one has to rely on algebraic means in order to solve all LP's problem in general. The graphical approach solves LP problems by identifying the extreme point of the feasible set and testing the objective function value at the extreme points. The one which yields the best value for the objective function is selected as the optimal solution. The Simplex algorithm does the same thing using purely algebraic means. The algebraic means is necessary in higher dimensional (i.e. $n \geq 3$) due to our inability to perceive the feasible region or the objective function geometrically. In higher dimension the objective and constraint functions are not line segments but hyper-planes (a geometrical concept of a plane in higher dimension) and the feasible region not a plane polygon but a simplex which is a region bounded by hyper-planes.

2.2.4 Simplex Algorithm

All LP problems are transformed into standard form before the solution of the problem could be ascertained. The standard form LP is obtained by adding to or subtracting from each inequality constraint superfluous variables respectively known as "slack" or "surplus" variables. A "slack" variable is a non-negative variable which when added to the left-hand-side (LHS) of a less-than-or-equal-to constraint transforms it into an equality constraint. A "surplus" variable on the other hand transforms a greater-than-or-equal-to constraint into an equality constraint. The simplex algorithm is described by the following steps:

- Putting the LP in standard form.
- Finding an initial basic feasible solution (IBFS)
- Checking whether the IBFS is optimal
- If IBFS is not optimal, find a new BFS with a better objective function value

- Repeating bullet three and four until there is no better value of the objective function, indicating that the current value was optima, or until it is clear that there is no optimal solution.

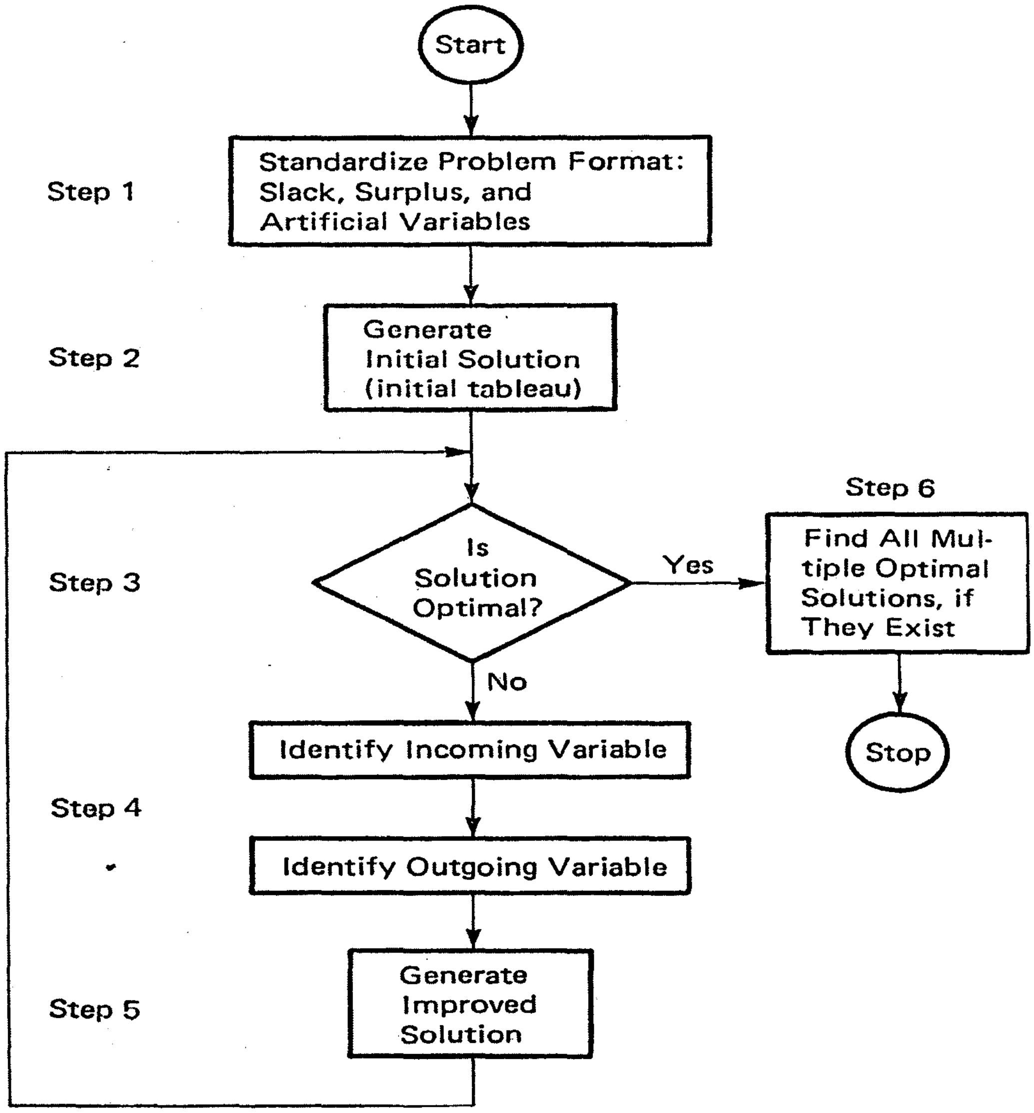

Figure 2.1 Schematic presentation of the Simplex method

2.2.5 Minimization versus maximization problems

Standard form LP problems consist of a maximizing or minimizing an objective function. Simplex method is described based on the maximization type of the standard form LP problems. However, if the objective functions is of minimization type, simplex method may still be applied with a small modification. The required modification can be done in either of the following two ways:

- The first approach required that we change the rule used to introduce a variable into the basis. In the maximization problems, we select the variable with the largest positive net evaluation value as the entering variable. To solve the minimization problems, we simply reverse this rule. That is, we select the variable with the most negative net evaluation value as the entering variable. We continuing the process until every value in the net evaluation row are zero or positive.
- The second approach to solving a minimization problem is by converting the problem into an equivalent maximization by multiplying the objective function by negative one (-1). Then solving the resulting maximization problem would provide the optimal solution to the minimization problem.

2.2.6 The Solution Algorithm

The Simplex method is an iterative search algorithm for large LP problems, starting from the initial ("origin", all $x = 0$) and moving toward adjacent "corner" points at the direction in which improvement on objective function value is maximized. If one "corner" point solution is better than all adjacent "corner" point solution, it is "optimal". The Simplex algorithm examines a finite number of basic feasible solutions (BFS) for optimality. For examines subset of the BFS for optimality. Even so, for very large size problems, the subset of BFS to examine could still be large or very large. Software packages of the Simplex algorithm abound today and can be implemented on a computer.

2.2.7 Integer Linear Programming (ILP): is one aspect of LP techniques with the additional requirement that some or all of the decision variables must be integer. An ILP where all the decision variables and possibly the objective function are required to be integer is called "all-integer linear program". However, when the word "integer" is dropped from model formulated in the context of optimization problems, it would lead to two kinds of models: (1) a model that result from dropping the integer requirements for the decision variables is called "Linear Programming Relaxation (LRP)" of the ILP and (2) model that restrict some but not all of the decision variables to be integer is called "Mixed Integer Linear Programming (MILP)"

There are numerous method use in solving integer linear programming problems but the most common one is the "Branch and Bound (BB or B&B)" method. This method is used to deal with problems either manually or computationally depending on the complexity of the problem before the analyst. The concept of the BB is to divide (branching) original large problem into smaller sub problems and bounding the best solution in the subsets. The algorithm for using this method is given below:

- Solve the problem without integer restrictions,
- If the solution is integer, then this must be the solution to integer problem,

- If these variables are not integer valued, the feasible region is divided by adding constraints restricting the value of one of the variables that was not integer valued,
- Bounds on the value of the objective function are found and used to help determine which sub-problems can be eliminated and when the optimal solution has been found,
- If a solution is not optimal, a new sub-problem is selected and branching continues.

A Branch-and-Bound algorithm consists of a systematic enumeration of all admissible solutions, where large subsets of fruitless candidates are discarded by using upper and lower estimated bounds of the quantity being optimized.

2.3 Concept of Media Selection Planning and Related Works

The media selection and planning problem has been stated in many ways. Essentially, the media planner must select a number of advertising media such as: newspapers, Television, Radio, Magazine etc. to carry out his ads over a predetermined period of time, and he must specify how many and which ads will be placed in each medium at each point in time that the medium is available. He must do this within a prescribed total advertising budget and in a way that will effectively expose some appropriate target audience to his advertising messages. The majority of approaches to the media selection planning problem seek to optimize some variant or others of the total advertising impressions provided by a schedule for a given approach. Definition of the problem varies according to the nature of the approach used, the variables included and the specification of the functions involved in the problem.

Optimization research related to media selection planning begun with the work done by Agostini, (1961) the contribution in this field was of much excitement to practitioners and advertising agencies, since it was a step forward for allocating advertising budget to media vehicles. Their criterion was to maximize the number of target exposure and they achieved this objective by solving a series of equations to arrive at the final value. Again, they tackled non-linear elements such as frequency discount on certain media, audience duplication among different media and successive insertions were cleverly handled by means of built-in statistical adjustment factors. Such factors tended to limit the model's initial tendency to build up too many insertions in the various media vehicles with the lowest impact weight.

Next to the pioneer model is that of Young and Rubican, (1963) who also came out with High Assay Media Model (HAMM) a media selection problem model. This model placed more emphases on the weekly performance of media schedules. For instance, magazines tend to accumulate their audiences over a period of weeks and these patterns are estimated based on special tabulations of early Politz magazine audience surveys which is an entity specialized in conducting advertising research so as to deliver the patterns of weight that are reflective of the seasonality of an entity business. The word High Assay in the name of the model places emphasis on the value of targeting and this is reflected in the notion that a small percentage of customers actually represent a large percentage of total business opportunities for any marketer. Thus, in today's business world, marketers' top priority is to find ways to identify and discriminatingly reach these high value target customers. This is normally in today's addressable advertising campaign.

Besides these earlier models, there are other scholars who made efforts to come out with other desirable media models to aid advertising executives and clients to make informed decisions. To quote one of the leading authors, "In essence, the complexity of the media problem has led to a variety of methods of solution depending on the assumptions made in each case" (Broadbent, 1966). For example advocates of the 'linear programming approach to the media selection and planning problem would favour the following problem statement offered by Day (1962): "Media selection problem is to allocate a scarce resource among a large number of alternatives so that the best possible contribution is made to a central objective". The objective to be maximized is the marketing effectiveness of the advertising programme, i.e. obtaining the maximum possible impact on the pertinent market target with a given budget. Nevertheless differences in definition exist even within those advocating similar approaches. For example, Bass and Lonsdale, (1966) in formulating their linear programming approach stated the problem as one of selecting the best set from among various media alternatives where alternatives are taken to include not only media but specific choices within medium. According to Stefanos et al., (2003) they investigated the media selection allocation problem to determine the optimum media for a commercial enterprise business entity. They adopted the WAA method as proposed by Bass and Lonsdole, (1966) together with circulation and readership data from the advertising agencies to achieve their desired objective. Day, (1962) however, talks of maximizing the 'marketing effectiveness', Bass and Lonsdale, (1966) proposed that the criterion for comparing alternative media selection and schedules is taken to be 'exposure'. A further variant on the selection of the maximization criterion is provided by Aaker, (1975) who argues that the aim of the media selection planning process is to determine within a given budget, that media insertion schedule which will obtain the greatest impact for an advertising campaign of given length. Furthermore, the contribution of Little et al., (1969) is also important. "Given a set of media options, a budget and various data about the media and the audience, which options should be used and when should they be used?" Such a statement of the problem differs from those earlier mentioned in that it implicitly recognizes the time element. The majority of media approaches to the media problem suffer in that they are static models apportioning the media budget to alternative media in order to maximize 'effectiveness' at some point in time. Several authors have pointed out this failing – "those media models of the media process must surely include the possibility of the decay of effect through time in the absence of reinforcement". In addition to the above, Stasch, (1965) made his view in regards to media selection from the erstwhile work done by Bass and Londole,(1966) They mentioned that for linear programming to attain its ultimate practical utility then available media data must be obtained at national, metropolitan, municipal levels, and even be broken down into age, sex, and income. Furthermore, he stated that the readership data and objective as well as constraints function must be translated into constant restraints, so as to reduce subjectiveness. Similarly Brown and Warshaw, (1965) contributions are worth mentioning. "Any conceptual model of the media process must surely include the possibility of the decay effect through time in the absence of reinforcement" Probably the most comprehensive treatment of the media selection problem is that of Lee, (1963) who from the outset goes beyond the usual media selection and schedule assessment criteria of coverage and impact in considering those campaigns which are intended to generate and maintain 'a specific level of awareness' in a predetermined target population throughout a certain period. This is not to suggest that work has not been carried out on the dynamic aspect of media response, merely that up until recently such treatments have been regarded as the prerogative of the advertising theorist and management scientist.

Stasch, (1965) showed how to develop a linear programming to incorporate additional space and time dimension in the model. The objective of the model was to minimize media cost in respect to audience exposure. The model was tailored towards to answer the questions, "When the advert should appear" and "Which market they should appear"? Further, he explained how the objective function of the model could be modified to avoid audience overlap and adding additional internal constraints to prevent understating the cost associated with the media selection process.

Shocker, (1970) stated that from the large number of feasible media schedules which can be constructed. In addition, from substantial discounts offered for multiple purchases of the same or related media vehicles and from interactions between media alternatives, which influence both the cost and effectiveness of different selections. Finally, it stems from the interrelations between the problem of allocating the advertising media budget and that of determining the size of that budget, the nature of the advertising theme and copy and the appropriate measures of effectiveness of advertising exposures. However, Chang et al., (2006) made their contribution in the field of media scheduling by looking critically at the earlier work done by Zufryden, (1973) the aim of the investigation was on the advertising wear out phenomenon. He finally proposed an optimal media mix plan on the advertising pulsation strategy to resolve the problem whilst maximizing the target exposure. Gensch, (1967) added that other factors include the climate of the media vehicle, its prestige, the media qualities in relation to the requirements of the product message, and the selection process. Akaaro, (2009) made his investigation into the media selection planning decision making process by stressing on the fact that awareness creation is the ultimate aim in any advertising campaign. He designed and implements an optimal media mix on an entire population on the spread of the epidemic HIV/AIDS in a country. A sample of the population was used for this advertising campaign after which a generalization was made on the whole country. Kotler, (1971) summarized some of the most important assumptions of linear programming as follows: (1) L.P. models assume that repeat exposures have a constant effect. (2) L.P. models assume constant media costs. (3) They cannot handle the problem of audience duplication.

2.4 Relevant Mathematical Programming Models

The following is a brief survey on media selection planning problems solved by other mathematical programming techniques not necessarily linear programming. This survey is intended to provide insights into some of the existing techniques and algorithms employed to tackle these problems.

Pagan and Moore, (2013) proposed a goal programming (GP) to design and implement a case study media mix plan problem for a private-physician's office. They sought to achieve two main objectives; (1) to maximize the yearly target exposure and (2) to minimize both annual and quarterly budget for the physician's office. To build the model using the above optimization technique, they looked at the number of ads placed in a specific local media groups (radio, newspapers, magazine, church bulletins, and publications) as well as internet listings. The audience exposure and cost data were surveyed for one month period, and then extrapolated on quarterly basis. The model was prescribed with eighteen (18) media vehicles and seventy-two (72) constraints variables. Since their priority was to maximize target exposure and minimize cost, they incorporated deviation variables into each of the budget constraints to allow for flexibility of the solution. Finally, the model was ascertained via the afore-mention optimization technique and the solution was compared with other existing model especially the qualitative and heuristic ones. The comparison showed that there was a vast difference between the existing models and the current one. Therefore, they concluded that their model could save much advertising dollars to the private-physician's office.

Aggarwal, (2012) developed and formulated a fuzzy multi-objective optimization problem in firm manufacturing consumer durables products which are to be marketed in segmented markets. The model was made up of three main media categories viz; print, website, and TV media with specific media vehicles in each category. The objective of the model was to select an appropriate media that will maximize the total advertising exposure and at the same time increase the customers' rate for the different products in the segmented markets. A numerical example was used which served as a stepping stone for solving the problems.

Cosgun *et al.*, (2012) described the challenges faced with clients in selecting advertising space to reach their prospective target customers in a TV network company. They looked at the factors hampering the media space selection such as; limited time, different rates for different target groups, and the relationship between a client and the TV network station. Regardless of such factors they developed an integer programming model for an advertisement campaign. The figure-of-merit which they were aiming at was to maximize the advertisement revenue. The formulated model was solved via Generalized Algebraic Modeling System (GAMS) software package and the solution of the model was of significance to the TV network station.

Maffei, (1960) proposed a dynamic programming approach to solve a media selection schedule problem of allocating a given advertising budget among three media in a test market. The approach went by almost unnoticed since it was very small and quite unrealistic. In line with the above models, a well known dynamic programming applied to media selection planning according to the literature is MEDIAC. It was developed by Little and Lodish (1966) and was later modified. They used sales as the criterion for the media schedule. They defined the sales result for the schedules as the sum of all the sales in each of a number of market segments over a number of time periods. They stated further that the sales in each segment in turns depended on the number of persons in the segment, their sales potential, and the level of advertising exposure in the segment. The sales were viewed as a response function exhibiting diminishing returns to the exposure value. The dynamic programming in MEDIAC is to maximize sales over planning period subject to advertising budget and media restrictions. They made the following assumptions which are the basis of the weakness in the model. Firstly, the model was limited to four time periods, since the nature of dynamic programming in combination with the storage capacity of computers does not permit longer time spans. Similarly, the model assumed that only one media class (magazine) and only fifteen (15) vehicles were relevant. These are not very practical when one considers the length of most major advertising campaigns and the number of media categories and vehicles available. In addition, the assumption that sale was a function of advertising exposure alone, meant that other determinants such as price, product quality, product availability, competitive behaviour and environmental factors had no significant impact on sales and this is ridiculous in today's advertising and promotional program.

In Lee and Burkart's, (1960) model for media selection planning, the objective was to maximize frequency and reach respectively. They developed mathematical relationship for the stated two goals. They attempted to maximize the frequency of the campaign via a heuristic rule of purchasing advertising in an inverse proportion to the square of the cost per thousand, whilst the maximization of reach was tackled under the assumption that the square of the proportion of the target group readership for a medium, divided by the cost of an insertion was constant for all media groups.

Taylor, (1963) developed graphical heuristic procedures to derive solution to the exposure problem formulated by Lee and Burkart's (1960). In his model, both the number of ads and size of insertion for each medium could be determined by finding the point (graphically) where the marginal returns of the last insertion equaled the cost of insertion for each medium. Finally Ellis,(1966) modified Lee and Burkart's (1960) work by the introduction of a more complete probabilistic response function by assuming different probabilities of exposure for different people in the target group.

Keown and Duncan, (1979) formulated an integer goal programming model for Magazine advertising campaign. They applied priority weights for each goal in the model. They accounted for factors such as audience overlap, discount, and maximum and minimum exposure levels in each of the goals of the model. The model was limited to only one media category advert campaign.

Deckro and Murdock, (1987) developed an integer linear programming for a multi-objective problem for various media choices such as TV, Radio, and Magazine etc. for an agency. The objective of the model was to optimize three reach functions for the chosen media categories considered by the agency. Data on reach for the various media alternatives were collected and imputed into the objective functions so as to ascertain the "trade-off" for the agency.

Mihiotis and Tsakiris, (2004) developed an algorithm to solve the integer programming problem of placing a commercial in a TV channels so as to achieve the largest TV viewing to the target customers. They achieved the objective of their research by optimally selecting combinations of TV spots based on the given TV-viewing ratings and budget limitations. The model was not extended to other media groups which we intended to investigate in this study.

Tektas and Alakavug, (2003) developed linear integer programming media model where the opportunity-to-see was maximized by giving weights using an analytical hierarchy process. The essence of the weights would allow the model to choose Magazine whose readers matched the profile of the targeted readers by the advert campaign. The model focused on only just one aspect of media categories along side with probabilities in the form of weights which aided qualitative decision in the media selection process.

Zangwill, (1965) made an attempt to overcome the problem of fractional linear programming (LP) solutions via integer linear programming (ILP). He used an integer programming model mainly to address the problem of integrality of the optimal and decision variables solution values in a Magazine advertising campaign. In addition, He considered other LP problems such as audience duplication in the media category but the model did not consider other media alternatives broadly which we intended to investigate in this study.

Renan, (2011) developed linear integer goal programming model for agencies to help them select bus exterior spaces in various campaign insertions in the transit media categories. The objective of the research was to optimize the number of insertions in the various media group considered yet working within the frame of budget limitations of the agency. The strength of the model was evaluated on how it can handle numerous campaigns for both the same or different customers and the target market coefficient was taken into account. The objective of the problem was ascertained with the help of A Mathematical Programming Language (AMPL) Solver. This model looked at a different aspect of media alternatives within the traditional mass media varieties and came out with the finding which was worth mentioning in the frontier of knowledge.

CHAPTER THREE

MODEL DEVELOPMENT

3.1 Introduction

Linear programming (LP) techniques of Operation Research (OR) can be used to formulate the problem of media selection for advertisement. The LP model shall be used to focus on where to advertise so as to increase exposure to the target customers. A common scenario of media selection planning problems arising in marketing companies and industries concern the allocation of a fixed advertising budget across various media types.

Every day, consumers are bombarded with different kinds of adverts on health conditions, Education, Hygiene, shopping, and drinking etc. All these listed advertisement are transmitted through the media such as Radios, Televisions, Newspapers, Billboards, and posters etc. The cost and target market (i.e. the prospective customers') response rate differs from each of these media used for advertisement. Again, an advert can take place in theatres by artists, posters or by cars. All these have their costs associated with them and their effectiveness in terms of target audience reached and the response rate can be elicited. The objective of this study is to determine the total target exposure for the various combination of media used in the advertisement campaign programme.

3.2 Sources of Data

The relevant data in regard to the cost per ad in each medium, the target exposure, the minimum and maximum ad contracted for each vehicle, and the total advertising budget for the promotion was obtained at the MTN headquarters with the aid of a questionnaire. Some of the data solicited, especially the target exposure was obtained at some of the various media outlets across the country. Also, the cost of buying a space or time in both the print and broadcast media was verified at the advertising agencies such as Graphic Communication Group (GCG), Ghana Broadcasting Corporation (GBC), and Media Monitoring Research Centre (MMRC) respectively. In addition, there were capacity restrictions on how to allocate the advertising budget to the various media groups. The objective of media selection planning problem is to maximize the combination of the target exposure for the various media categories.

3.3 Formulation of the Media Selection Planning Problem

Most marketing companies conduct advertisement to reach their customers with their product or to inform them about the service they offer. The most important question they seek to answer is how much of the advertising budget should be allocated to each of the various media group in order to maximize the overall target exposure for the company. Usually, companies also face many constraints on the availability of the inputs they use in their investment activities and management policies. The problem is then to determine the media mix that will maximize the total target exposure subject to the input constraints it faces. The formulation is done as follows:

- The objective function to be optimized is express as a linear function.

- The firm's constraints are also expressed as linear inequalities. (The reason is that the firm can often use up to, but not more than, specified amounts to be invested, or the firm must meet some minimum requirement).
- In addition, there are nonnegative constraints on the decision variables to indicate that the firm cannot produce a negative output or use a negative quantity of any input.
- Find the optimal solution (i.e., the values of the decision variables) at the extreme point or corner of the feasible region that touches the highest isoprofit line. This represents the optimal solution to the problem subject to the constraints faced.

3.4 Data Analysis Tools

Software Packages exist for the implementation of the Simplex method in a computer towards solving linear optimization problems. These Software Packages vary by their degree of flexibility, user friendliness, ability to handle large scale problem, and computational time to output the result. Some of these tools are; LP Solver, Microsoft Excel Solver and Quantitative Manager (QM). Thus, at the end of formulating the model, any of these software packages can be used to solve the LP problem that would be formulated from the data gathered. Any one of these gives the same output of analysis. However the Quantitative Manager was used for the analysis due to its simplicity and accuracy.

3.5 Sensitivity Analysis

Sensitivity analysis is very useful when attempting to determine the impact the actual outcome of a particular variable will have if it differs from what was previously assumed. By creating a given set of scenarios, the analyst can determine how changes in one variable will impact the target variable. Different LP packages have different formats for input/output but the same information as discussed above is still obtained. Essentially the interpretation of LP output is something that comes with practice. Much of the information obtainable (as discussed above) as a by-product of the solution of the LP problem can be useful to management in estimating the effect of changes (e.g. changes in costs, production capacities, profit margins, etc) without going to the hassle/expense of resolving the LP. This sensitivity information gives us a measure of how robust the solution is i.e. how sensitive it is to changes in input data. Conducting a sensitivity analysis is beneficial in several ways. Not only can one make better and more informed decisions by changing assumptions and observing, or estimating the results, one is also better able to predict the outcome of his decisions. For example, if sensitivity analysis is conducted before deciding to increase prices, the decision is less risky than if one did not go through this exercise. In addition to proving with a glimpse into the future, sensitivity analysis leads to faster decisions.

3.6 Description of the Problem

The Mobile Telecommunication Network (MTN) Company Ghana is launching an advertisement promotional campaign dubbed; "MTN Hitmaker Season 2" which is intended to last for three months. The programme is aimed at identifying young artists who wanted to become the best musician in the country. The contestants were required to drop their demos at the various MTN branches across the country. Also, the aspirants had the opportunity to take part in a nationwide selection process thus; the "want-to-be musicians" are required to go to some selected radio stations in all the Regional Capitals to sing their songs upon which listeners and viewers will call or text to vote for their favorite. The best in all the regions together with other contestants in the Capital City of Ghana will be invited to the MTN headquarters to vie for the final and runner-up positions respectively.

Since this programme is targeting all who want to be the best musician in the country and to inform the general public as well, the advertiser needs to advertise the programme to the general public and the prospective target audiences to inform them of the existence of the programme. This can only be accomplished by buying space or time on the various media to achieve the advertiser's desired objective. Currently, there are a lot of mass media in the country and the advertiser may not utilize all to maximize the target exposure due to financial constraint. This necessitates that they must opt for some media alternatives to bring about exposure. The advertiser has decided to use some selected state owned and private media to achieve the desired objective. The media are categorized into three major media types viz; TV, Radio, and Newspapers with specific selected media vehicles within each category since prospective target audiences have choices to which media they used to obtain information.

3.7 Data for the Problem

Data was collected on the media categories which the MTN intended to use for the programme. These are as follows:

- Television Media:

GTV, TV3, Metro TV, Viasat 1, and Crystal TV

- Print Media:

Daily Graphic, Graphic Showbiz, Ghanaian Business, Junior Graphic, Ghanaian Sports, The Mirror, Daily Guide, The Chronicle, Ghanaian Times, Ninety Minutes

- Radio Media:

Adom FM, Hitz FM, Joy FM, Happy FM, Peace FM, Asempa FM, Radio Gold, Citi FM, Unique FM, BBC Radio, Choice FM, Hot FM, Obonu FM, Angel FM, Hello FM, Love FM, Kapital Radio, Nyira FM, Otec FM, Kessben FM, Radio Central, Sunrise FM, Kyzz FM, Aseda FM, Melody FM, Volta Star, Savana Radio, Uria Radio, Radio Upper West (UW), Radio Bar Thus five (5) TV media, ten (10) Print media, and thirty (30) Radio media were used for the adverts.

- The total advertising budget fixed for the promotional campaign was GH¢ 600,000.00
- The budget allocation to TV media was GH¢200, 000.00
- The maximum budget allocation to the Print media was 25% of the total budget (i.e 150,000).
- The budget allocation to Radio media was GH¢ 250,000.00
- The advertiser's policy was that advert should be cast on all the seven days of the week with priority on working days and Saturdays.

- The number of advert that should be cast in any of the broadcast media should be at least three (3) times and the maximum advert available in any of the broadcast media was 270 throughout the length of the program.
- The advertiser's policy was that the Advertising units on Radio media should be more than that of TV.
- The advertiser's policy was that the Advertising units on Radio media should be more than that of Print.
- The advertiser's policy was that the Advertising units on TV media should be more than that of Print.
- The advertiser's policy was that the maximum budget allocation to Radio media should be more than that of TV.
- The advertiser's policy was that the maximum budget allocation to Radio media should be more than that of Print.
- The budget allocation to TV media should be more than that of print.
- The advertiser expects that the promotional campaign reaches a greater number of audiences, and yet work within its budgetary limit. Therefore it has estimated that a total of 500 adverts may be cast across all media. The policies and restrictions given above would constitute the major constraints of the model.

3.7.1 Print Media Advert Availability

In considering the number of times to use the various print media, the supply is on daily basis and as such an advertiser expects to see a full page or part of a page per day. The available advert in the various Newspapers for the print media considered in this problem is based on the number of times the Newspaper is produced for the length of the program. The minimum restriction on the Newspaper for the advertiser is to expect at least one (1) advert in each of the print media and the maximum is no more than the number of times a particular Newspaper is produced for the period of the advertising campaign. Table 3.4 shows the maximum available advert for the various print media which are considered in this media selection problem.

Table 3.1 Cost of advert and Estimated Audience Exposure for TV Media

Television media	Cost (GH¢) per advert	Estimated Audience Exposure
Metro TV	780	447,599
GTV	900	895,199
Viasat 1	500	413,169
TV3	427	516,461
Crystal TV	590	103,292

Table 3.2 Cost of advert and Estimated Audience Exposure for Print Media

Print Media	Cost(GH¢) per advert	Estimated Audience Exposure
Daily Graphic	3,928.12	2,000,000
Graphic Showbiz	970.70	500,000
Ghanaian Business	1,614.72	900,000
Junior Graphic	812.67	750,000
Ghanaian Sports	970.70	700,000
The Mirror	2,213.64	1,600,000
Daily Guide	2,909.50	30,000
The Chronicle	1,648	550,000
Ghanaian Times	1,203.93	600,000
Ninety Minutes	682.12	650,000

Table 3.3 Cost of advert and Estimated Audience Exposure for Radio Media

Radio Media	Cost (GH¢) per advert	Estimated Audience	Radio Media	Cost (GH¢) Per advert	Estimated Audience
Adom FM	103	65,655.84	Kapital Radio	90	11,197.34
Hitz FM	68	32,827.92	Nyira FM	25.9	78,381.38
Joy FM	151	87,541.12	Otec FM	42	55,986.7
Happy FM	25.9	32,827.92	Kessben FM	50	45,565.42
Asempa FM	42	21,885.28	Radio Capital	25	25,888.94
Radio Gold	35.7	32,827.92	Sunrise FM	98	21,885.28
Citi FM	45	21,885.28	Kyzz FM	40	31,074.48
Unique FM	62	3,884.31	Aseda FM	35	3,305.86
BBC Radio	87	3,884.31	Melody FM	25	23,305.86
Choice FM	76.8	10,942.64	Volta Star	95	10,942.64
Hot FM	45	32,827.92	Savana Radio	25	21,885.28
Obonu FM	48.7	10,942.64	Uria Radio	25.9	20,597.15
Angel FM	48.7	3,884.31	Radio UW	69	16,798.87
Hello FM	83.9	34,368.08	Radio Bar	45	98,651.56
Love FM	64	67,184.04	Peace FM	42	28,279.2

Table 3.4 Print Media advert availability per Length of Campaign

Print Media	Maximum advert
Daily Graphic	90
Graphic Showbiz	12
Ghanaian Business	36
Junior Graphic	24
Ghanaian Sports	12
The Mirror	12
Daily Guide	90
The Chronicle	12
Ghanaian Times	90
Ninety Minutes	36

3.8 Mathematical Model Formulation

In this section, a linear programming formulation of the media selection planning of the MTN Company is developed. The decision variables, objective function and constraints of the problem are also presented.

3.9 The Decision Variables

The decision variables are the various media outlets from which the advertiser must choose to develop the advertising campaign. The decision variables consist of the number of times to use each medium in order to maximize the desired target exposure whilst operating under the restrictions and policies of the company. Since all the media outlets considered in this study fall into three major media categories, the decision variables are made up of five (5) from TV, ten (10) from print and thirty (30) from radio media. The decision variables are defined as:

X_{ti}: Number of TV adverts

$(i = 1,2, \dots ,5)$ (t i.e index for TV)

X_{pj}: Number of Print adverts

$(j = 1,2, \dots ,10)$ (p i.e index for Print)

X_{rk}: Number of Radio adverts

$(k = 1,2, \dots ,30)$ (r i.e index for Print)

3.10 The objective Function

The objective of the problem is to find how to allocate the advertising budget across the various media so as to maximize the total target audience exposure for the campaign, whilst operating under the budgetary and policy restrictions presented earlier. Buying a unit space or time in each media is to a certain number of audience exposures. Therefore audience exposure associated with each media is a parameter of the objective function. Let (A_{ti}, A_{pj}, A_{rk}) and let (X_{ti}, X_{pj}, X_{rk}) where A_{ti}, A_{pj}, A_{rk} are respectively row vectors for TV, Print, and Radio audience exposures X_{ti}, X_{pj}, X_{rk} representing the decision variables.

The objective function can thus be expressed in the form;

$$\text{Maximize } Z = AX \tag{1}$$

Where

$$AX = \sum_{i=1}^{5} A_{ti} X_{ti} + \sum_{j=1}^{10} A_{pj} X_{pj} + \sum_{k=1}^{30} A_{rk} X_{rk} \tag{2}$$

Thus equation (2) can be expressed in a matrix notation as

$$AX = \sum_{l=1}^{3} \left(\sum_{i,j,k=1}^{5,10,30} A_{mn} X_{mn} \right) \tag{3}$$

Where

$$m = t, n = i, \forall\ i = 1,2, \ldots ,5$$

$$m = p, n = j, \forall\ j = 1,2, \ldots ,10$$

$$m = r, n = k, \forall\ k = 1,2, \ldots ,3$$

$$l \in \{t, p, r\}$$

The objective function is thus:

$$\text{Maximize } Z = \sum_{l=1}^{3} \sum_{i,j,k=1}^{05,10,30} A_{mn} X_{mn} \tag{4}$$

3.11 The constraints

The constraints of the problem could broadly be classified into two groups; the first group dealt with the budget allocation to the respective media and their relationship between them. The next group also dealt with the number of adverts available for each media category per the length of the advertising campaign. The policies and restrictions on the budget allocation and advert requirement as presented in section 3.7 was converted into the constraints as given below.

3.11.1 The Budget constraints

Let (C_{ti}, C_{pj}, C_{rk}) represent row vectors for TV, Print, and Radio cost of advert respectively. Let (X_{ti}, X_{pj}, X_{rk}) denotes the decision variables of the problem whilst maintaining the definition of the other indices; the total budget constraint is developed in a similar manner as in section 3.10 above.

- Total budget for campaign program: $\sum_{l=1}^{3}\sum_{i,j,k=1}^{5,10,30} C_{mn}X_{mn} \leq 600{,}000$ (5)
- TV Media

  Total Budget: $\sum_{i=1}^{5} C_{t\,i}X_{ti} \leq 200{,}000$ (6)

  TV Media budget in relation to Print Media budget:

  $\sum_{i=1}^{5} C_{t\,i}X_{ti} - \sum_{j=1}^{10} C_{p\,j}X_{pj} > 0$ (7)
- Print Media

  Total budget: $\sum_{j=1}^{10} C_{p\,j}X_{p\,j} \leq 150{,}000$ (8)
- Radio Media

  Total budget: $\sum_{r=1}^{30} C_{r\,k}X_{rk} \leq 250{,}000$ (9)

  Radio Media budget in relation to TV Media:

$$\sum_{r=1}^{30} C_{r\,k} X_{rk} - \sum_{i=1}^{5} C_{t\,i} X_{ti} > 0 \quad (10)$$

Radio Media budget in relation to Print Media:

$$\sum_{r=1}^{30} C_{r\,k} X_{rk} - \sum_{j=1}^{10} C_{p\,j} X_{pj} > 0 \quad (11)$$

3.11.2 Media Advert Constraints

- TV Media:

Maximum advert for each TV media: $X_{ti} \leq 270 \forall i$ (12)

Available TV adverts in relation to Print: $\sum_{i=1}^{5} X_{t\,i} - \sum_{j=1}^{10} X_{p\,j} > 0$ (13)

- Print Media:

Maximum adverts for each TV media:

$X_{pj} \leq \{90, 12, 36, 24, 12, 12, 90, 12, 90, 36\} \; \forall j$ (14)

- Radio Media:

Maximum adverts for each Radio media: $X_{rk} \leq 270 \quad \forall k$ (15)

Available Radio adverts in relation to TV: $\sum_{k=1}^{30} X_{r\,k} - \sum_{i=1}^{5} X_{t\,i} > 0$ (16)

Available Radio adverts in relation to Print: $\sum_{k=1}^{30} X_{r\,k} - \sum_{j=1}^{10} X_{p\,j} > 0$ (17)

- Total adverts per day across all media type: $\sum_{l=1}^{3} \sum_{i,j,k=1}^{5,10,30} X_{mn} \geq 500$ (18)

The resulting LP model has 45 decision variables and 56 constraints and is given by equations 4 to 18 above.

CHAPTER FOUR

COMPUTATIONAL RESULTS AND SENSITIVITY ANALYSIS

4.1 Results of the LP model

The LP was run in the first instance on a PC using QM (Quantitative Manager) Software version 2.2 by Chang and Sullivan (1987). The results indicated that some of the values of the decision variables were non-integer hence the LP solver version 5.5.2.0 which made provision for an integer requirement was used to solve the model. The result of the LP is summarized in Table 4.1. The optimal values of the decision variables are integers which was the desired optimal solution for the advertising promotional program.

Table 4.1 Results of LP Model

Television		Radio			
Variable & Media Outlets	**# of adverts**	**Variable & Media Outlets**	**# of adverts**	**Variable & Media Outlets**	**# of adverts**
X_{t1} : Metro TV	0	X_{r16}: Adom FM	270	X_{r31}: Kapital Radio	0
X_{t2} : GTV	143	X_{r17}: Hitz FM	270	X_{r32} : Nyira FM	270
X_{t3} : Viasat 1	1	X_{r18}: Joy FM	270	X_{r33} : Otec FM	270
X_{t4} : TV3	165	X_{r19}: Happy FM	269	X_{r34} : Kessben FM	270
X_{t5} : Crystal TV	0	X_{r20}:Asempa FM	270	X_{r35}: Radio Capital	270
Print					
X_{p6} : Daily Graphic	3	X_{r21}: Radio Gold	270	X_{r36} : Sunrise FM	0
X_{p7} : Graphic Showbiz	24	X_{r22}: Citi FM	270	X_{r37} : Kyzz FM	270
X_{p8}:Ghanaian Business	12	X_{r23}: Unique FM	0	X_{r38} : Aseda FM	0
X_{p9} :Junior Graphic	12	X_{r24}: BBC Radio	0	X_{r39} : Melody FM	270
X_{p10} : Ghanaian Sports	36	X_{r25}: Choice FM	0	X_{r40} : Volta Star	0
X_{p11} : The Mirror	12	X_{r26}: Hot FM	270	X_{r41}: Savana Radio	270
X_{p12} : Daily Guide	0	X_{r27}: Obonu FM	0	X_{r42} : Uria Radio	270
X_{p13} : The Chronicle	0	X_{r28}: Angel FM	0	X_{r43} : Radio UW	0
X_{p14} : Ghanaian Times	0	X_{r29}: Hello FM	2	X_{r44} : Radio Bar	270
X_{p15} : Ninety Minutes	35	X_{r30}: Love FM	270	X_{r45} : Peace FM	270
Optimal Value = 541,399,986					

4.2 Interpretation of the Results

The objective function value was 541,399,991.3800 which was the greatest number of target audience exposure for the length of the advertising campaign program. The objective of the media selection program was to maximize the total target audiences' exposure per the length of the promotional program whilst operating under the policies and restrictions that would be identified in the marketing firm.

The output of the model indicates that advertisement should be made 143 times in GTV, 165 times in TV3, and a single broadcast in Viasat 1 was enough to yield the overall exposure for the Television media categories. For the newspapers, advertisement should be made 12 times each in Ghanaian Business, Junior Graphic, and The Mirror whilst advertisement should be made 36 times in Ghanaian Sports, 24 times in Graphic Showbiz, and 35 times in Ninety Minutes as indicated by the basis variables in the output of the model in table 4.1 above. In addition, advertisement should be made 3 times in Daily Graphic throughout the length of the program. For the radio stations, advertisement should be made 270 times in each of Adom FM, Love FM, Joy FM, Peace FM, Citi FM etc. as indicated by the basis variables in the solution given above. These values showed that advertisement should be maintained constantly in respect to the above radio stations throughout the period of the programme. However, advertisement should be made 2 times in Hello FM which means that it has a small contribution to the overall optimal value. The basis variables having a solution at zero (0) at optimum level are non-economical to use, though estimated target exposure values in some of them may be encouraging such as Unique FM and Angel FM but they are non- optimal decision variables. Finally, the slack or surplus solutions at optimality informed the decision maker that either there is more of advertising space or time in a particular media or the advertiser has to secure extra advertising space or time in some of the media to run the advertising campaign at the expense of sacrificing resources to achieve the intended objective.

Therefore, the recommended optimum media mix for the promotional campaign program was made up of each candidate media categories. These include three (3) Television outlets, seven (7) newspaper, and twenty (20) radio stations respectively. These media mix can be achieved without sacrificing any budget.

4.3 Sensitivity Analysis

Sensitivity analysis on a linear programming model provides information on the sensitivity of the optimal solution when data values are changed. When formulating a linear programming model, data is often estimated based on experience and assumptions and can therefore be inaccurate. Sensitivity analysis in linear programming offers extensive capability to demonstrate how the optimal solution changes when the data values are changed. This would help make better recommendations and reduce errors in making decisions.

Sensitivity analysis for the media selection planning problem seeks to answer the following specific questions: (1) How will a variation in the number of audience exposures in any of the media categories affect the optimal solution?

(2) How will a variation in the budget allocation to the respective media categories affect the optimal value?

(3) How will a variation in both the budget and the audience exposure affect the optimal solution?

Also, the redundancy of a constraint was put to the same test, so that we check the effect of the solution on the optimal value.

Since the solution of the model was obtained through the branch and bound method which make it possible to arrive at integer values for the model. Thus, a meaningful and automated sensitivity analysis could not be obtained directly. Therefore, it is necessary to revise and submit the model multiple times whilst varying one or more parameters in order to check the solution effect on the optimum solution.

In order to answer the above questions, the estimated target exposure values for each media categories was decreased and increased by 10% and the resulting LP problem was solved and the solution was compared with the original LP problem.

4.3.1 Analysis of TV Media Exposures up and down by 10%

As the TV exposures were up and down by 10%, each of the exposure values of the original LP were calculated by the same amount whilst maintaining the decision variables in the other constraints constant. The parameter values in the revised model were imputed in the MILP solver. The output of the results is presented in Table 4.2 below.

Table 4.2 Results of LP for the Variation on the TV Media Exposure

Television		**Radio**			
Variable & Media Outlets	**# of adverts**	**Variable & Media Outlets**	**# of adverts**	**Variable & Media Outlets**	**# of adverts**
X_{t1} : Metro TV	0	X_{r16}:Adom FM	270	X_{r31}:Kapital Radio	0
X_{t2} : GTV	143	X_{r17}: Hitz FM	270	X_{r32} : Nyira FM	270
X_{t3} : Viasat 1	1	X_{r18}: Joy FM	270	X_{r33} : Otec FM	270
X_{t4} : TV3	165	X_{r19}:HappyFM	269	X_{r34}:Kessben FM	270
X_{t5} : Crystal TV	0	X_{r20}:Asempa FM	270	X_{r35}:Radio Capital	270
Print					
X_{p6} : Daily Graphic	3	X_{r21}:Radio Gold	270	X_{r36}: Sunrise FM	0
X_{p7} : Graphic Showbiz	24	X_{r22}: Citi FM	270	X_{r37} : Kyzz FM	270
X_{p8} :Ghanaian Business	12	X_{r23}:Unique FM	0	X_{r38} : Aseda FM	0
X_{p9} :Junior Graphic	12	X_{r24}:BBC Radio	0	X_{r39}: Melody FM	270
X_{p10} : Ghanaian Sports	36	X_{r25}:Choice FM	0	X_{r40} : Volta Star	0
X_{p11} : The Mirror	12	X_{r26}: Hot FM	270	X_{r41}:Savana Radio	270
X_{p12} : Daily Guide	0	X_{r27}: Obonu FM	0	X_{r42} : Uria Radio	270
X_{p13} : The Chronicle	0	X_{r28}: Angel FM	0	X_{r43} : Radio UW	0
X_{p14} : Ghanaian Times	0	X_{r29}: Hello FM	2	X_{r44} : Radio Bar	270
X_{p15} : Ninety Minutes	35	X_{r30}: Love FM	270	X_{r45} : Peace FM	270
Increased Optimal Value = 562,764,255					
Decreased Optimal Value = 520,035,717					

The results of varying the TV media exposures for both up and down by 10% shows that; as the estimated target exposures were decreased by 10% in the TV media category, the expected optimal value dropped from 541,399,986 to 520,035,717 and this is about 21,364,269 lose in optimal value which corresponds to less than4%of the original value. The reduction in the optimal value may be due to general lose in viewership of the advertising programs that are used to telecast the promotional program to the prospective target audiences. Again, as the estimated target exposures were increased by 10% for the same media set, the optimal value increased from 541,399,986 to 562,764,255 and this is about 21,364,269 gains in optimal value which correspond to almost 4%. The gain in optimal value may be due to the advertising agencies telecasting the promotion in programs that catches the interest of the prospective target audiences. The output of the revised model is the same as that of the original model – the solution of the decision variables do not change in value and this can be verified in table 4.1 above. The difference only occurred in the optimal values of the revised models.

4.3.2 Analysis of Print Media Exposures up and down by 10%

As the print exposures were up and down by 10%, each of the exposure values of the original LP were calculated by the same amount whilst maintaining the decision variables in the other constraints constant. The parameter values in the revised model were imputed in the MILP solver. The output of the results is presented in Table 4.3.

Table 4.3 Results of LP for the Variation on the Print Media Exposure

Television		Radio			
Variable & Media Outlets	**# of adverts**	**Variable & Media Outlets**	**# of adverts**	**Variable & Media Outlets**	**# of adverts**
X_{t1} : Metro TV	0	X_{r16}: Adom FM	270	X_{r31} : Kapital Radio	0
X_{t2} : GTV	143	X_{r17}: Hitz FM	270	X_{r32} : Nyira FM	270
X_{t3} : Viasat 1	1	X_{r18}: Joy FM	270	X_{r33} : Otec FM	270
X_{t4} : TV3	165	X_{r19}: Happy FM	269	X_{r34} : Kessben FM	270
X_{t5} : Crystal TV	0	X_{r20}:Asempa FM	270	X_{r35}: Radio Capital	270
Print					
X_{p6} : Daily Graphic	3	X_{r21}: Radio Gold	270	X_{r36} : Sunrise FM	0
X_{p7} : Graphic Showbiz	24	X_{r22}: Citi FM	270	X_{r37} : Kyzz FM	270
X_{p8} :Ghanaian Business	12	X_{r23}: Unique FM	0	X_{r38} : Aseda FM	0
X_{p9} :Junior Graphic	12	X_{r24}: BBC Radio	0	X_{r39} : Melody FM	270
X_{p10} : Ghanaian Sports	36	X_{r25}: Choice FM	0	X_{r40} : Volta Star	0
X_{p11} : The Mirror	12	X_{r26}: Hot FM	270	X_{r41} : Savana Radio	270
X_{p12} : Daily Guide	0	X_{r27}: Obonu FM	0	X_{r42} : Uria Radio	270
X_{p13} : The Chronicle	0	X_{r28}: Angel FM	0	X_{r43} : Radio UW	0
X_{p14} : Ghanaian Times	0	X_{r29}: Hello FM	2	X_{r44} : Radio Bar	270
X_{p15} : Ninety Minutes	35	X_{r30}: Love FM	270	X_{r45} : Peace FM	270
Increased Optimal Value = 551,894,986					
Decreased Optimal Value = 530,904,986					

The LP results of varying the print media exposures for both up and down by the same amount of 10% indicated a dropped from 541,399,986 to 530,904,986 when the exposures values were decreased by 10% for the print media category. This is about 10,495,000 reductions in exposure and this is approximately 2% of the expected value. The reduction in exposure values may be due to the fact that the advertising agencies have not placed the promotion in advertising programs that will arouse the interest of the prospective target audiences. The result also shows that the optimal value increased from 541,399,986 to 551,894,986when the exposure values were increased by the same amount in the print media category. The increment in exposure values gain about 10,495,000 exposures and this is approximately 2% of the original value. The margin in the exposure value may be due to the promotion been cast in advertising programs that catches the interest of the target audiences.

The output of the revised model is the same as the original model – the solution of the decision variables do not change in values and this can be verified in table 4.1 above. The difference only occurred in the optimal values of the revised models.

4.3.3 Analysis of Radio Media Exposures up and down by 10%

As the radio exposures were up and down by 10%, each of the exposure values of the original LP were calculated by the same amount whilst maintaining the decision variables in the other constraints constant. The parameter values in the revised model were imputed in the MILP solver. The output of the results is presented in Table 4.4 below.

Table 4.4 Results of LP for the Variation on the Radio Media Exposure

Television		**Radio**			
Variable & Media Outlets	**# of adverts**	**Variable & Media Outlets**	**# of adverts**	**Variable & Media Outlets**	**# of adverts**
X_{t1} : Metro TV	0	X_{r16}: Adom FM	270	X_{r31} : Kapital Radio	0
X_{t2} : GTV	143	X_{r17}: Hitz FM	270	X_{r32} : Nyira FM	270
X_{t3} : Viasat 1	1	X_{r18}: Joy FM	270	X_{r33} : Otec FM	270
X_{t4} : TV3	165	X_{r19}: Happy FM	269	X_{r34} : Kessben FM	270
X_{t5} : Crystal TV	0	X_{r20}: Asempa FM	270	X_{r35}: Radio Capital	270
Print					
X_{p6} : Daily Graphic	3	X_{r21}: Radio Gold	270	X_{r36} : Sunrise FM	0
X_{p7} : Graphic Showbiz	24	X_{r22}: Citi FM	270	X_{r37} : Kyzz FM	270
X_{p8} :Ghanaian Business	12	X_{r23}: Unique FM	0	X_{r38} : Aseda FM	0
X_{p9} :Junior Graphic	12	X_{r24}: BBC Radio	0	X_{r39} : Melody FM	270
X_{p10} : Ghanaian Sports	36	X_{r25}: Choice FM	0	X_{r40} : Volta Star	0
X_{p11} : The Mirror	12	X_{r26}: Hot FM	270	X_{r41} : Savana Radio	270
X_{p12} : Daily Guide	0	X_{r27}: Obonu FM	0	X_{r42} : Uria Radio	270
X_{p13} : The Chronicle	0	X_{r28}: Angel FM	0	X_{r43} : Radio UW	0
X_{p14} : Ghanaian Times	0	X_{r29}: Hello FM	2	X_{r44} : Radio Bar	270
X_{p15} : Ninety Minutes	35	X_{r30}: Love FM	270	X_{r45} : Peace FM	270
Increased Optimal Value = 563,680,721					
Decreased Optimal Value = 519,119,257					

The LP result of varying the radio media exposures for both up and down by the same amount shows that the maximum target exposure dropped from 541,399,986 to 519,119,257 as the exposure values were decreased in that category. The reduction in the exposure values lead to a loss of about 22,280,729 exposures which corresponds to approximately 4% of the expected value. The reduction in exposure values could be due to the advertising agencies losing listenership in programs that will attract the interest of the prospective target audiences.

Also, as the exposure values were increased in the radio media category, the figure-of-merit increased from 541,399,986 to 563,680,721 and this increment is about 22,280,735 exposures which correspond to approximately 4% of the original value. The extra gain in exposure values could be due to the advertising agencies reclaiming their listenership audiences after positioning themselves well in the advertisement competitive environment.

The output of the revised model is the same as the original model – the solution of the decision variables do not change in value and this can be verified in Table 4.1 above. The difference only occurred in the optimal values of the revised models.

4.3.4 Analysis of Audience Exposure up and down by 10%

As the audience exposures were up and down by 10%, each of the exposure values of the original LP were calculated by the same amount whilst maintaining the decision variables in the other constraints constant. The parameter values in the revised model were imputed in the MILP solver. The output of the results is presented in Table 4.5.

Table 4.5 Results of LP for the Variation on the Audience Exposure

Television		Radio			
Variable & Media Outlets	**# of adverts**	**Variable & Media Outlets**	**# of adverts**	**Variable & Media Outlets**	**# of adverts**
X_{t1} : Metro TV	0	X_{r16}: Adom FM	270	X_{r31} : Kapital Radio	0
X_{t2} : GTV	143	X_{r17}: Hitz FM	270	X_{r32} : Nyira FM	270
X_{t3} : Viasat 1	1	X_{r18}: Joy FM	270	X_{r33} : Otec FM	270
X_{t4} : TV3	165	X_{r19}: Happy FM	269	X_{r34} : Kessben FM	270
X_{t5} : Crystal TV	0	X_{r20}: Asempa FM	270	X_{r35}: Radio Capital	270
Print					
X_{p6} : Daily Graphic	3	X_{r21}: Radio Gold	270	X_{r36} : Sunrise FM	0
X_{p7}: Graphic Showbiz	24	X_{r22}: Citi FM	270	X_{r37} : Kyzz FM	270
X_{p8}:Ghanaian Business	12	X_{r23}: Unique FM	0	X_{r38} : Aseda FM	0
X_{p9} :Junior Graphic	12	X_{r24}: BBC Radio	0	X_{r39} : Melody FM	270
X_{p10} : Ghanaian Sports	36	X_{r25}: Choice FM	0	X_{r40} : Volta Star	0
X_{p11} : The Mirror	12	X_{r26}: Hot FM	270	X_{r41} : Savana Radio	270
X_{p12} : Daily Guide	0	X_{r27}: Obonu FM	0	X_{r42} : Uria Radio	270
X_{p13} : The Chronicle	0	X_{r28}: Angel FM	0	X_{r43} : Radio UW	0
X_{p14} : Ghanaian Times	0	X_{r29}: Hello FM	2	X_{r44} : Radio Bar	270
X_{p15} : Ninety Minutes	35	X_{r30}: Love FM	270	X_{r45} : Peace FM	270
Increased Optimal Value = 595,539,974					
Decreased Optimal Value = 487,259,993					

The revised model of increasing the entire cost coefficient by 10% indicated a significant increase in the optimal value when the values were increased. The increase in the entire cost coefficient made the optimal value to increase from 541,399,986 to 595,539,974 this appreciable change in the optimal value corresponds to a gain of 54,139,988which is equivalent to 10%of the original value.

Again, as the cost coefficients were decreased in exposure values, there was a drastic reduction in the optimal value. The maximum target exposure dropped from 541,399,986 to 487,259,993when the cost coefficients were reduced by 10%. The reduction in all the exposure values make the optimal value to reduce by about 54,139,993 exposures and this is equivalent to 10% of the optimal value.

The output of the revised model is the same as the original model – the solution of the decision variables do not change in value and this can be verified in Table 4.1. The difference only occurred in the optimal values of the revised models.

4.3.5 Analysis of Budgetary constraints up and down by 10%

The discussion of the post analysis test on this aspect is in two fold. We shall first looked at the upward variation on the right hand side of the budgetary constraints and then followed by the downward variation on the same stated parameters by a certain amount and the results shall be provided as well for each scenario. As the budgetary constraints were varied upward by 10%, the maximum target exposure increases from 541,399,986 to 578,780,665 in the revise model.

The increment in the budgetary constraints gained about 37,380,679 exposures and this is equivalent to less than 7% of the original value. In addition, increasing the budgetary constraints by the same percentage makes some of the media mix variables to enter into the basis to become basic variables and vice versa. For example, Radio Upper West entered the basis to become basic variables thus, creating a more media mix for the advertising campaign. Moreover, increasing the budgetary constraints makes some of the basis variables to improve in their solution value which leads to the overall margin in the maximum target exposure value. The solution values of GTV, TV3, and Daily Graphic were improved in solution value as a result of the upwards adjustment in the budgetary constraints. The above analysis on the parameter values can be verified by juxtaposing their solution values from the original model and the revised model results. Notwithstanding these alterations in the budgetary constraints, some of the basic and non – basic variables still maintain their solution values in the reformulated model.

Table 4.6 Results of LP for the Variation on the Budgetary Constraints up by 10%

Television		Radio			
Variable & Media Outlets	**# of adverts**	**Variable & Media Outlets**	**# of adverts**	**Variable & Media Outlets**	**# of adverts**
X_{t1} : Metro TV	0	X_{r16}: Adom FM	270	X_{r31} : Kapital Radio	0
X_{t2} : GTV	165	X_{r17}: Hitz FM	270	X_{r32} : Nyira FM	270
X_{t3} : Viasat 1	1	X_{r18}: Joy FM	270	X_{r33} : Otec FM	270
X_{t4} : TV3	166	X_{r19}: Happy FM	270	X_{r34} : Kessben FM	270
X_{t5} : Crystal TV	0	X_{r20}: Asempa FM	270	X_{r35}: Radio Capital	270
Print					
X_{p6} : Daily Graphic	7	X_{r21}: Radio Gold	270	X_{r36} : Sunrise FM	0
X_{p7} : Graphic Showbiz	24	X_{r22}: Citi FM	269	X_{r37} : Kyzz FM	270
X_{p8} :Ghanaian Business	12	X_{r23}: Unique FM	0	X_{r38} : Aseda FM	0
X_{p9} :Junior Graphic	12	X_{r24}: BBC Radio	0	X_{r39} : Melody FM	269
X_{p10} : Ghanaian Sports	36	X_{r25}: Choice FM	0	X_{r40} : Volta Star	0
X_{p11} : The Mirror	12	X_{r26}: Hot FM	270	X_{r41} : Savana Radio	270
X_{p12} : Daily Guide	0	X_{r27}: Obonu FM	0	X_{r42} : Uria Radio	270
X_{p13} : The Chronicle	0	X_{r28}: Angel FM	0	X_{r43} : Radio UW	37
X_{p14} : Ghanaian Times	0	X_{r29}: Hello FM	270	X_{r44} : Radio Bar	270
X_{p15} : Ninety Minutes	34	X_{r30}: Love FM	270	X_{r45} : Peace FM	270
Optimal Value = 578,780,665					

Similarly, as the budgetary constraints were varied downward by 10%, the optimal value dropped from 541,399,986 to 502,271,807 in the revised model. The reduction in the budget constraints decreased the optimal value for about 39,128,179 lost in exposure and this corresponds to less than 7% of the original value. Also, the downward variation in the budget constraints by the same degree of percentage made some of the media mix variables to leave the basis to become non-basic and vice versa. For example, Daily Graphic, Hitz FM, and Hello FM left the basis to become non-basic variables thus, leading to a less media mix for the advertising campaign programme. Again, the reduction in the budget constraints made some of the basis variables to reduce in solution value which lead to the overall margin in the objective functional value in the revised model. The values of GTV, Citi FM and, Graphic Showbiz reduced in solution values as a result of the downwards adjustment in the budgetary constraints. The analysis of these parameter values can be verified from the original and revised models result respectively.

Table 4.7 Results of LP for the Variation on the Budgetary Constraints down by 10%

Television		Radio			
Variable & Media Outlets	**# of adverts**	**Variable & Media Outlets**	**# of adverts**	**Variable & Media Outlets**	**# of adverts**
X_{t1} : Metro TV	0	X_{r16}: Adom FM	270	X_{r31} : Kapital Radio	0
X_{t2} : GTV	120	X_{r17}: Hitz FM	0	X_{r32} : Nyira FM	270
X_{t3} : Viasat 1	1	X_{r18}: Joy FM	270	X_{r33} : Otec FM	270
X_{t4} : TV3	167	X_{r19}: Happy FM	270	X_{r34} : Kessben FM	270
X_{t5} : Crystal TV	0	X_{r20}: Asempa FM	270	X_{r35}: Radio Capital	270
Print					
X_{p6} : Daily Graphic	0	X_{r21}: Radio Gold	270	X_{r36} : Sunrise FM	0
X_{p7} : Graphic Showbiz	21	X_{r22}: Citi FM	126	X_{r37} : Kyzz FM	270
X_{p8} :Ghanaian Business	12	X_{r23}: Unique FM	0	X_{r38} : Aseda FM	0
X_{p9} :Junior Graphic	12	X_{r24}: BBC Radio	0	X_{r39} : Melody FM	270
X_{p10} : Ghanaian Sports	36	X_{r25}: Choice FM	0	X_{r40} : Volta Star	0
X_{p11} : The Mirror	12	X_{r26}: Hot FM	270	X_{r41} : Savana Radio	270
X_{p12} : Daily Guide	0	X_{r27}: Obonu FM	0	X_{r42} : Uria Radio	269
X_{p13} : The Chronicle	0	X_{r28}: Angel FM	0	X_{r43} : Radio UW	0
X_{p14} : Ghanaian Times	0	X_{r29}: Hello FM	0	X_{r44} : Radio Bar	270
X_{p15} : Ninety Minutes	35	X_{r30}: Love FM	270	X_{r45} : Peace FM	270
Optimal Value = 502,271,807					

4.3.6 Redundancy of a Constraint

The LP has 45 decision variables and 56 constraints thus, leading to 56 equations with 45 unknown variables. In solving linear programming problems, the number of decision variables must equal the number of constraints function in the model. It follows that some of the constraints may not contribute to the optimal value of the LP problem. This will render those constraints redundant and can therefore be ignored for the effectiveness of the model.

4.3.7 Analysis of the model without the constraint "at least 500 adverts cast across all media group"

The cost coefficient of the objective function and the technological coefficient of the constraint functions of the LP problem are imputed into the MILP Solver software used to determine the output. The results of the output were the same as that of the original LP problem – the optimal value was 541,399,986 - as shown in Table 1.4. Since there is no alteration in the solution values of the decision variables or the objective function value, it suffices that the above constraint is redundant and the analyst can do without it. The model was revised several times to check out all possible redundant constraints.

4.3.8 Audience Exposure up and Budget Allocation down by 10%

As both audience exposure and budget allocation values were up and down by 10%, each of the audience exposure and budget allocation values of the original LP were calculated by the same amount whilst maintaining the decision variables in the other constraints constant. The parameter values in the revised model were imputed in the LP solver.

The output of the revised model was the same as that of Table 4.7 above – the solution of the decision variables do not change in values and this can be verified in Table 4.7 earlier mentioned. The difference only occurred in the optimal value of the revised model.

From the results, the solution value of the decision variables is the number of times to use any of the media outlet associated with it to achieve the overall maximum target audience exposure. It is observed from the results that the expected optimal value increased from 541,399,986 to 552,498,977 resulting in a difference of 11,098,991 which corresponds to 2% of the original optimal value. Also, the variations in both the audience exposure and budget allocation by the same degree of percentage made some of the media mix variables to leave the basis to become non-basic and vice versa. For example, Daily Graphic, Hitz FM, and Hello FM left the basis to become non-basic variables thus, leading to a less media mix for the advertising campaign programme. Again, the alterations in the afore-mentioned units made some of the basis variables to reduce in solution value which lead to the overall margin in the objective functional value in the revised model. The values of GTV, Citi FM and, Graphic Showbiz reduced in solution values as a result of the adjustment made in them.

Notwithstanding these variations, some of the decision variables still maintain their solution values which imply that they had a tremendous impact in the final optimal value. The analysis of these parameter values can be verified from the original and revised models result respectively.

4.3.9 Audience Exposure down and Budget Allocation up by 10%

As both audience exposure and budget allocation values were down and up by 10%, each of the audience exposure and budget allocation values of the original LP were calculated by the same amount whilst maintaining the decision variables in the other constraints constant. The parameter values in the revised model were imputed in the LP solver.

The output of the revised model was the same as that of Table 4.6 above – the solution of the decision variables do not change in values and this can be verified in Table 4.6 earlier mentioned. The difference only occurred in the optimal value of the revised model.

From the results, the solution value of the decision variables is the number of times to use any of the media outlet associated with it to achieve the overall maximum target audience exposure. It is observed from the results that the expected optimal value dropped from 541,399,986 to 520,902,609 resulting in a deficit of 20,497,377 which corresponds to less than 4% of the original optimal value. In addition, the variations in both the audience and the budget allocation values by the same percentage made some of the media mix variables to enter into the basis to become basic variables and vice versa. For instance, Radio Upper West entered the basis to become basic variables thus, creating a more media mix for the advertising campaign. Moreover, the alterations in the audience exposure and the budget allocation values made some of the basis variables to improve in their solution values which lead to the overall margin in the maximum target exposure value. The solution values of GTV, TV3, and Daily Graphic were improved in solution values as a result of the adjustment in them.

Notwithstanding these variations, some of the decision variables still maintain their solution values which imply that they had a tremendous impact in the final optimal value.

The analysis of these parameter values can be verified from the original and revised models result respectively.

4.4 Summary of results and sensitivity Analysis

The results as in Table 4.1 showed that the recommended media mix for the study includes some of each candidate media which was collectively given as three (3) from Television, seven (7) from print, and twenty (20) from radio media outlets. These recommended media mix identified would generate an optimal value of 541,399,986 target audience exposure as the objective that the company was intended to achieve within the period of the advertising campaign programme.

Also, sensitivity analysis test was conducted on some of the parameter values and their resulting models were compared with the original LP problem. The comparison of the results as presented in table 4.8 shows that they all proposed some media outlets within each media category

Next, the right – hand – side (RHS) of the budgetary constraints values was varied at 10% level, so as to gauge the effect on the objective function value. In the first scenario, each of the RHS of the budgetary constraints values was decreased by 10% and this lead to a corresponding reduction in the optimal value to 502,271,807 exposures per the length of the programme. Conversely, an increased in the RHS of the budgetary constraint values by the same percentage increased the optimal value to 578,780,665 exposures.

An interesting observation that was made in the RHS budgetary values variation was that new variables either entered or left the basis which lead to the creation of new media mix for the advertising campaign programme. Again, some of the basis variables increased or decreased in solution values which lead to a subsequent effect on the optimum value.

Further, table 4.8 shows that increasing each of the media category exposure values one at a time by 10% would lead to a corresponding increased in the objective function value, likewise decreasing each of the media category exposure values by the same percentage would lead to a reduction in the objective function value with their respective percentages.

Furthermore, it was observed that as all the audience exposures values were varied by 10% upward, the objective function value increased significantly to 595,539,974 exposures and this corresponds to about 54,139,988 increments in the optimal value. Similarly, as all the audience exposures were varied downward by 10%, the objective function value decreased drastically to 487,259,993 exposures and this corresponds to about 54,139,993 lost in the optimal value. The solution values of the decision variables were the same as compared with the original LP problem.

Moreover, all the audience exposure and the RHS budgetary constraints values were subjected to varied up and down by 10%, the results of the revised model shows that the objective function value increased to 552,498,977 exposures and this corresponds to an increment of about 11,098,991 exposures in the optimal value. On the contrary, as all the audience exposures and the RHS budgetary constraints values were varied down and up by the same percentage, the resulting objective function value decreased to 520,902,609 which corresponds to a reduction of about 20,497,377 exposures in the optimal value. Lastly, the redundancy of any constraint in the model was also analyzed to see whether there might be any changes in the optimal value.

All these variations in the model notwithstanding, some of the media outlets such as Ghanaian Business, Junior Graphic, Ghanaian Sports, The Mirror for the print media group, Joy FM, Happy FM, Peace FM, Radio Bar etc for the radio media group, and GTV, Viasat1, TV3 for the television media group always remained in the basis regardless of any variation in the model. The presence of these media outlets in the basis implied that they had a tremendous impact on the optimal value.

In sum, by revising the model on some selected parameters, we are able to determine the impact those parameters would had on the objective function value so that we could make useful recommendation to the company. Sensitivity analysis enables the decision maker to make informed decisions to changes in a dynamic world. The sensitivity analysis was conducted on some selected parameters of interest but it could be done for any or all of the parameters upon which the model was built.

Table 4.8 Summary of Sensitivity Analysis

Media Category & Budget	Optimal Value	Optimal Val. up by 10%	Optimal Val. down by 10%
TV Media Group	541,399,986	562,764,255	520,035,717
Difference	0	21,364,269	21,364,269
% Value	0	4%	4%
Print Media Group	541,399,986	551,894,986	530,904,986
Difference	0	1,0495,000	10,495,000
% Value	0	2%	2%
Radio Media Group	541,399,986	563,680,721	519,119,257
Difference	0	22,280,729	22,280,735
% Value	0	4%	4%
Audience Exposure Val.	541,399,986	595,539,974	487,259,993
Difference	0	54,139,988	54,139,993
% Value	0	10%	10%
Budget Values	541,399,986	578,780,665	502,271,807
Difference	0	37,380,679	39,128,179
% Value	0	7%	7%

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The study considered a media selection planning for MTN Company in Ghana with the aim of maximizing the target audience exposure whilst operating under the policies and restraints of the company. The research took into account the various media outlets considered for the advertising campaign programme which includes; five (5) Television media, ten (10) print media, and thirty (30) radio media in Ghana. Linear programming techniques was used to model the media selection problem in order to come out with a practical results that would aid the company in decision making process when it comes to buying advertising space or time in the media outlets.

5.2 Conclusions

In the process of formulating the media selection planning problem for the MTN Company, we attempted to simulate the actual operation as closely as possible in order to produce a realistic model that would be used for the purchasing of advertising space or time in the various media outlets considered in the research. The output of the model was ascertained by using LP programming techniques in conjunction with computer software packages. It was found that out of the three major media categories which give a total of Forty-Five (45) media outlets in the country and advertising budget of Six Hundred Thousand Ghana cedis (GH¢ 600,000), the optimal target audience exposure was

541,399,986. The optimum media mix which generated the desire objective value for the advertising campaign include; three (3) Television media outlet, seven (7) newspapers within the print media outlet, and twenty (20) FM stations within the radio media outlet respectively. Thus, Linear programming techniques has aided us to come out with effective media selection planning for the MTN Company.

By conducting sensitivity analysis test on the model for various parameters option, the MTN company can pre – assess the effect of a given parameters on the overall optimal value. The sensitivity analysis of the model shows that some media outlets such as Ghanaian Business, Junior Graphic, Ghanaian Sports, The Mirror for the print media group, Joy FM, Happy FM, Peace FM, Radio Bar for the radio media group, and GTV, Viasat1, TV3 for the television media group always remained in the basis regardless of any variation in the model. The presence of these media outlets in the basis implied that they had a tremendous impact on the optimal value.

Moreover, the availability of sufficient computer software such as QM and LP Solver had facilitated the generation of the output of the model easily and within a very short period of time. In addition, the proposed model can serve as a useful tool not only for marketing companies but also media practitioners, stakeholders, institutions, and even individuals for negotiation cost of adverts in the various media outlets in the country.

5.3 Recommendations

One of the most successful and important application of linear programming to solving business problems has been in the area of media selection planning. From the conclusions, it was realized that using quantitative methods to identify the best media mix for advertising companies has helped them to yield the desire audience exposure which will in turn bring about a turn over to the company.

It is therefore recommended that the MTN company should focus on advertising on some of each candidate media outlets; three(3) television, seven(7) newspapers, and twenty(20) radio stations respectively for the budget allocation on the major media categories in order to optimize the target audience exposure for the length of the programme.

Also, I recommended that apart from identifying the best media mix for the advertising campaign programme for just one of the telecommunication company in the country, advertising institutions and other marketing companies should adapt scientific and mathematical approach in most of their advertising campaign programme.

In addition, it is recommended that the telecommunication company should adapt this model in the allocation of the advertising budget set for media selection planning problem. Further, marketing companies should be educated to employ quantitative method to find an appropriate quantitative model to assist them in the allocation of advertising budget more efficiently across the various media categories.

In this problem only a single criterion was optimized which is the audience exposure to reach by the company. In any advertisement promotional campaign, management would expect that the advertising budget which is the fixed cost be as small as possible. Therefore, the fixed cost could instead be minimized whilst the audience exposure is maximized in that perspective. Hence, it suffices that a bi – criterion of the media selection planning problem involving these factors as objective to be optimized simultaneously and this could provide a useful insight into the subject area. These lines of investigation would be the subject of future work.

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APPENDICES

Appendix 1: Budgetary Constraints up and down by 10%

Budget & Media Categories	Original Value	Value Up by 10%	Value Down by 10%
Total Budget	600,000	660,000	540,000
TV Media	200,000	220,000	180,000
Print Media	150,000	165,000	135,000
Radio Media	250,000	275,000	225,000

Appendix 2: TV Media Exposures up and down by 10%

Television Media	Original Exp. Value	Exp. Val. Up by 10%	Exp. Val. Down by 10%
Metro TV	447599	492359.9	402839.1
GTV	895199	9884718.9	805679.1
Viasat 1	413169	454485.9	371852.1
TV3	516461	568107.1	464814.9
Crystal TV	103292	113621.2	92962.8

Appendix 3: Print Media Exposures up and down by 10%

Print Media	**Original Exp. Value**	**Exp. Val. Up by 10%**	**Exp. Val. Down by 10%**
Daily Graphic	2,000,000	2,200,000	1,800,000
Graphic Showbiz	500,000	550,000	450,000
Ghanaian Business	900,000	990,000	810,000
Junior Graphic	750,000	825,000	675,000
Ghanaian Sports	700,000	770,000	630,000
The Mirror	1.600,000	1,760,000	1,440,000
Daily Guide	3,000	33,000	27,000
The Chronicle	550,000	605,000	495,000
Ghanaian Times	600,000	660,000	540,000
Ninety Minutes	650,000	715,000	585,000

Appendix 4: Radio Media Exposures up and down by 10%

Radio Media	**Original Exp. Val**	**Exp. Val. Up by 10%**	**Exp. Val. Down by 10%**	**Radio Media**	**Original Exp. Val.**	**Exp. Val. Up by 10%**	**Exp.Val. Down by 10%**
Adom FM	65,655.84	72,221.424	59,090.256	Kapital Radio	11,197.34	12,317.074	10,077.606
Hitz FM	32,827.92	36,110.712	29,545.128	Nyira FM	78,381.38	86,219.518	70,543.242
Joy FM	87,541.12	96,295.232	78,787.008	Otec FM	55,986.7	61,585.37	50,388.03
Happy FM	32,827.92	36,110.712	29,545.128	Kessben FM	45,565.42	50,121.94	41,008.86
Asempa FM	21,885.28	24,073.808	19,696.752	Radio Capital	25,888.94	28,477.834	23,300.046
Radio Gold	32,827.92	36,110.712	29,545.128	Sunrise FM	21,885.28	24,073.808	19,696.752
Citi FM	21,885.28	24,073.808	19,696.752	Kyzz FM	31,074.48	34,181.928	27,967.032
Unique FM	3,884.31	4,272.741	3,495.879	Aseda FM	3,305.86	3,636.446	2,975.274
BBC Radio	3,884.31	4,272.741	3,495.879	Melody FM	23,305.86	25,636.446	20,975.274
Choice FM	10,942.64	12,036.904	9,848.376	Volta Star	10,942.64	12,036.904	9,848.376
Hot FM	32,827.92	36,110.712	29,545.128	Savana Radio	21,885.28	24,073.808	19,696.752
Obonu FM	10,942.64	12,036.904	9,848.376	Uria Radio	20,597.15	22,656.865	18,537.435
Angel FM	3,884.31	4,272.741	3,495.879	Radio UW	16,798.87	18,478.757	15,118.983
Hello FM	34,368.08	37,804.91	30,931.29	Radio Bar	98,651.56	10,8516.76	88,786.44
Love FM	67,184.04	73,902.444	60,465.639	Peace FM	28,279.2	31,107.12	25,451.28