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JOSHUA MICHAEL BUNYAN

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BY

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THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, FACULTY OF PHYSICAL SCIENCES, UNIVERSITY FOR DEVELOPMENT STUDIES IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTER OF PHILOSOPHY DEGREE IN MATHEMATICS

JULY, 2025

DECLARATION

Student

I hereby declare that this thesis is the result of my original work and that no part of it has been presented for another degree in this University or elsewhere. Related works by others which served as a source of knowledge has been duly referenced and acknowledged.

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Supervisor

I hereby declare that the preparation and presentation of this thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University for Development Studies.

Supervisor's Signature…………………………… Date 16/07/2025

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ABSTRACT

The small-scale agricultural sector makes a substantial contribution to a country's economic growth. The goal of this research is to create a multi-objective optimization model for farm planning that maximize returns whilst minimizing labour cost for optimal land use. To do this, three optimization models were developed using the weighted sum and epsilon-constraint method for multi-objective programming and solved by optimization techniques, using a management scientist software version 6.0. These three models were discussed in detailed and tested using data collected from some selected small-scale vegetables, cereals and legume farmers in some parts of Navrongo in the Kesena Nankana Municipality of the Upper East region of Ghana. Model 1 suggested an optimal land use and an increased in returns for all the test it was employed on. Also model 2 showed an increased in returns in all test cases while suggesting an optimal land use as well. Furthermore, model 3 showed a decreased in cost of employing labour whiles suggesting an optimal land usage. In all cases, the models were found to be robust for both the vegetables, cereal and legume farming through the sensitivity analysis done for the range of values for the coefficients of the decision variables and the constraints. In general, the developed models helped the small-scale farmers to maximize returns as well as minimizing the cost of labour whilst proposing optimal use of the farm land. Consequently, it is recommended that future works could consider adding other parameters like fertilizers application or type of fertilizer used, soil type, water requirements, etc. to see how the model will work.

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Lastly, to all who has contributed to this work in diverse ways, I express my gratitude and ask God to abundantly reward all of your hard work.

DEDICATION

I dedicate this thesis to my new born son Judah Godslove Bunyan and the family at large.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ABBREVIATIONS

CMOEA - Constrained Multiobjective Evolutionary Algorithms

CMOP - Constrained Multiobjective Optimization Problems

FAO - Food and Agriculture Organization

GDP – Gross Domestic Product

LP – Linear Programming

MCDA – Multi-Criteria Decision Analysis

MOE – Ministry of Education

MOFA – Ministry of Food and Agriculture

MOIP – Multi-objective Integer Programming

MOLP – Multi-objective Linear Programming

MOMIP – Multi-objective Mixed Integer Programming

MOO – Multi-objective Optimization

MOODSS – Multi-objective Optimization Decision Support System

MO-MINLP – Multi-objective Mixed-Integer Linear Programming

NSGA- II - Non-dominated Sorting Genetic Algorithm II

SPEA2 - Strength Pareto Evolutionary Algorithm 2

SSA – South Saharan Africa

WMM – Weighted Metric method

WSM – Weighted Sum Method

CHAPTER ONE

INTRODUCTION

1.0 Background to the study

Planning is very important in everyday life and for that matter individuals, institutions and several entities engage in planning for their daily routine. According to MAIB (2015), there are several definitions for farm planning which include; the process to allocate scarce resources of the farm, organize farm production in such a way as to minimize the utilization of scarce resources and to maximize the income of the farmer. Also, farm planning constitutes a systematic approach to decision-making or selecting from a range of competing alternatives. This process encompasses the numerous modifications that a farmer implements within the current organizational framework, aiming to optimize the utilization of limited resources for maximum profitability. According to Metcalf (1969) production decisions facing a farmer may be broadly divided into three groups such as a resource-product problem, resource-resource problem and product-product problem. Farm planning and controlling are concerned with the organization of the farmer's resources which includes, land, labour, machinery, buildings and breeding livestock and the management of the enterprises undertaken by the farm, such as milk, beef and cereal production (Doyle 1990). According to Barnard and Nix (1999), a farmer is considered as the manager or the business owner of a farm who carries out decision making activities.

Farm planning is useful for several reasons including enabling the farmer;

- attain his goals in a more structured way concerning his family and farm.
- examine existing resources and their best allocation.

- make choices about the type and quantity of livestock to be kept, the hectarage under various crops, and the crop selection.
- identify the input and credit needs.
- estimate future cost and returns.

In general, agriculture is the most important sector of Ghana’s economy, employing more than half the population on formal and informal basis and accounting for almost half of the country’s GDP and export earnings (Ministry of Food and Agriculture report, 2023). The nation cultivates a diverse array of agricultural products across multiple climatic regions, which extend from arid savanna to humid forest, and which traverse the country in east-west orientations. Among the agricultural commodities that constitute the foundation of Ghana's economy are yams, cereals, cocoa, oil palms, cashews, and timber.

Agriculture plays a central role in promoting growth and poverty reduction in Ghana. Ghana needs an agricultural revolution based on productivity growth. This will raise many Ghanaians out of poverty, improve rural livelihoods significantly, and make a dent in the poverty in many Ghanaian communities, especially in the five (5) Northern regions. In Ghana, agriculture contributes 35% and 45% of the country’s GDP and export earnings respectively and employing about 55% of its work force (GSS, 2012). Agricultural activities play very important roles through employment, poverty reduction, food security and enhancing the standard of living by increasing income levels of the rural population. The World Bank (2010) report shown that the increased in GDP derived agriculture was average, 2.9 times more effective in increasing the incomes of the poor in the developing countries. Irz *et al.* (2001) conducted research on agriculture productivity growth and poverty alleviation and reported that poverty in Africa saw a reduction of 7% from an estimate of every 10% increase in farm output.

In the context of Sub-Saharan Africa (SSA), agriculture constitutes the predominant source of sustenance, economic livelihood, and generation of foreign exchange revenue. Agricultural production plays a pivotal role in ensuring food security within SSA; consequently, the enhancement of agricultural productivity is essential for SSA's capacity to fulfill its food security and economic development aspirations amid the challenges posed by rapid population expansion. Sub-Saharan Africa is projected to have the highest population growth rate over the next decade, at 2.5% per year, with nearly three quarters of its workforce employed in agriculture (World Bank, 1998, Amos 2012). In SSAs, agriculture is faced with numerous challenges, including political instability, inadequate infrastructure and poverty.

Other factors that have reduced agricultural output in SubSaharan Africa include the ava ilability and quality of agricultural education and the difficulty in obtaining conventiona l inputs.

Furthermore, since 1990, Africa's fertilizer application rate has been falling at an averag e rate of 1.1% annually. When this trend is reversed and fertilizer usage is increased by 5% per year could increase agricultural output by an additional 0.5% per year (FAO, 2000).

Since agriculture is a significant economic sector in SSA, increasing agricultural output is a key policy objective. A sizeable section of the populace in all developing nations depends on this industry for both direct and indirect income, particularly in rural areas where poverty is more severe. As a result, an expanding agricultural sector helps to reduce poverty and promote general growth.

The economy of the Upper East region of Ghana which the study focuses is based largely on agriculture with about 70% of the population engaged in agricultural production. However, agriculture in the region is beset with single and erratic rainfall patterns which lead to poor crop yields. A progressive decline in the average level of rainfall has been observed (Assan *et al.,* 2009). Daze (2007) reports an estimated 74% decrease in rainfall by 2030. This makes the population's poverty position worse. Also 34% of the region's population, which is rural is food insecure; the highest in the Ghana and an additional 13% are vulnerable to food insecurity (WFP, 2009). Leahi (1988) also showed that areas with arid and semi-arid climates, the luck of uncertainty about rainfall would strongly be pointed to irrigation as a prime candidate to support future food strategies in the medium and long term. Likewise, Dessalegn (1999) suggests that, where there is insufficient and unreliable rainfall, rain-fed agriculture cannot fully support food production, investment on irrigation will help stabilize agricultural production and promote food security. Rain-fed agriculture is therefore no longer a viable option for the region's sustainable agricultural output. Therefore, irrigation development is crucial for improving the region's rural livelihoods and for producing agricultural in a sustainable manner.

Due to the fact that farming is the primary source of income and the primary occupation for the majority of rural households, agricultural growth has been crucial in reducing poverty.

For those who are impoverished, the significance is especially greater than for those who are not. While income from crop production has been declining over time between 1992 and 2006, agriculture still provides more than or close to 50% of total income for most rural households (GFA, 2010). Therefore, it is very important for critical attention to be given to agriculture to make it very lucrative and interesting for more individuals to come on board to help provide for the growing population and the world at large. Therefore, efficient method of farm planning is very important in achieving a good yield or maximizing agriculture production.

Early studies used mathematical programming techniques, specifically linear programming which revolves on the use of formal algorithms to select optimum solutions in farming (Swanson, 1956; Tyler, 1958). Heady and Dilion (1961) asserted that the analysis of static, single input-output relationships began to be replaced by more complex mathematical models capable of determining the most profitable allocation of resources between various farms. Formulating farm optimization models is associated with planning more than any other conventional management functions (Gurmesa, 2011). Farm models assist how resources should be allocated in farming activities. This study is an extension of early studies on application of linear programming in farming activities to include multi-objective optimization approach to farm planning which is rare in literature.

1.1 Statement of the problem

Despite the recent economic challenges in Ghana, the country has experienced substantial economic growth and a decline in poverty and hunger since the 1990s. However, this development has not been experienced equally across the country. Today, there is a dramatic north-south divide where poverty, as well as food and nutrition insecurity remain widespread in the northern savannah (IFAD, 2012). In the five Northern regions

(Northern, North East, Savannah, Upper East and Upper West) making up the northern savannah, the prevalence of poverty is at 52 to 88%, compared to around 30% in the Brong-Ahafo (Bono, Bono East and Ahafo) and Volta regions, and 12 to 20% in the five Southern regions (Greater Accra, Ashanti, Central, Eastern and Western) (Al-Hassan *et al.,* 2009).

Numerous developmental studies conducted in Ghana concluded that improved agricultural production is necessary to bring northern Ghana out of poverty (Shepherd, 2005, Diao *et al.,* 2007, Al-Hassan and Poulto*,* 2009). Hunger and poverty have many different and intricate causes. Agricultural development in the north is hindered by societal problems such insufficient infrastructure, a lack of economic possibilities, government corruption, and inadequate health and education. Increasing agricultural production is further complicated by environmental problems like pest pressures, harsh and unpredictable weather patterns, and soil sterility and degradation. The lack of access to cutting-edge agricultural technologies and farmers' poor understanding of better agronomic production and management techniques exacerbates these difficulties. Lastly, there is a notable disconnect between research, education, and extension as a result of poor communication and cooperation between the Ministry of Food and Agriculture (MOFA) and the Ministry of Education (MOE). As is clear, this disparity has improved small-scale farmers' livelihoods while posing serious challenges to agricultural development. However, to support equal development, several European countries, Canada and the United States continue to support pro-poor development projects in Ghana's northern regions (CIA, 2013; USAID, 2013).

However, every crop season, the majority of farmers must make difficult decisions. It should be mentioned that well-thought-out choices regarding which crop to plant, how to grow it, and how much to grow it in can effectively lead to enhanced agriculture. These farmers' choices are influenced by the current financial and physical limitations on their farms. Small-scale farmers in Ghana and other developing nations base their decisions on experience, gut feeling, and neighbour comparisons. These decisions made by the small-scale farmers lack budgeting techniques and comparative analyses which are key ingredients, useful for making farming decisions (Hazel & Norton, 1986).

The small-scale farmers in Navrongo, Ghana are faced with significant challenges in making optimal farm planning decisions due to limited access to advanced farming techniques and decision-support tools. These farmers often rely on intuition and traditional methods, which do not effectively address the multiple, competing objectives of maximizing profit, minimizing costs, and ensuring sustainability. There is a need for a multi-objective optimization model that can assist these farmers in making informed decisions that balance agricultural productivity, resource efficiency, farm outcomes and ultimately improving their livelihoods. Thus, this research seeks to formulate a multi-objective optimization model that will help the small- scale farmers to optimize land use, returns and labour cost.

1.2 Main objective

The main objective of this study is to formulate multi-objective optimization model for farm planning in small-scale farming.

1.2.1 Specific objectives

The specific objectives of this study are to:

1. Develop a multi-objective optimization (MOO) model for farm planning to maximize returns whiles minimizing labour cost.
2. Modify and solve the MOO model using the weighted sum method and Epsilon-constraint methods.
3. Carry out sensitivity analysis on the modified models developed.
4. Propose a more suitable solution procedure for optimal decision making.

1.3 Significance of the study

The study is of great important to the government, policy analysts, NGO’S and other institution/organizations and individuals in agricultural sector on how to apply efficient farm planning methods for good yield. The results of the study can be used by the small-scale farmers in Ghana to efficiently utilize the scares resource of farm land as well as minimize labour cost whiles maximize returns on farming.

Also, the study's conclusions will serve as the foundation as a basis for the Ministry of Food and Agriculture to promote sustainable agriculture and thriving agribusiness through research and technology development, effective extension and other support services to farmers, processors and traders for improved livelihood.

Again, farmers will benefit from the study's conclusions when making decisions, irrespective of the prevailing physical and financial farming constraints. It is also going to provide immense guide to individuals especially with regards to those who wish to carry out research in similar areas especially regarding appropriate research procedure.

1.4 Justification of the study

In the face of growing pressure on agricultural resources, Small- scale farmers in Ghana specifically in rural areas such as Navrongo, are repeatedly challenged to take a complex decision concerning crop selection, resource apportion, and input usage. These decisions must seldom balance multiple contention objectives like maximizing returns, minimizing cost, and safeguarding resources.

The conventional routine of farm planning typically centers on single objectives or rely on heuristic approach, which are insufficient for capturing the full complexity of and trade-offs inherent in agricultural frameworks. This study is defended by the urgent need to espouse scientific data-driven approaches, that reflect the real-world entanglements of small-scale farming.

Particularly, the use of multi-objective optimization (MOO) strategies like the Weighted Sum Method and the Epsilon Constraint Method provides a robust underlying structure to handle competing goals contemporaneously. These methods allow for the generation of set of optimal solutions (Pareto optimality), offering farmers flexible as well as informed decision-making options based on their preferences and constraints.

The application of these two MOO strategies is specifically notable. The Weighted Sum Method is intuitive and authorizes decision-makers to assign importance to every set objective. That is to say that, making it appropriate for participatory planning where farmers could express their priorities.

The Epsilon Constraint Method on the other hand is proficient of exploring non-convex regions of the Pareto front and it is usable when certain goals like minimum subsistence income or water conservation limits, must be satisfied strictly. Together, these models could provide a more wide-raging and adaptable planning tool than traditional linear or single -objective optimization strategies.

Furthermore, contributes to both theoretical knowledge and practical innovation in agricultural planning. It will serve as a worthful resource for agricultural extension services, development agencies, and policymakers seeking to promoting sustainable and efficient farming practices. It again aligns with Ghana's broader goals of improving food security, maximizing farm productivity, reducing rural poverty and also contributes to the achievement of several Sustainable Development Goals (SDGS), including SDG1 (No poverty), SDG 2 (Zero hunger), and SDG 12 (Responsible Consumption and Production).

In essence, this study gives a timely and important approach to transform small-scale agriculture in Navrongo via the application of advanced multi-objective decision-making models, which are both scalable and adaptable to other regions facing similar problems.

Additionally, the research advances both theoretical understanding and practical innovation in agricultural planning. It will be a key resource for agricultural extension services, development of organizations, and policymakers looking to promoting sustainable and efficient farming strategies. It also aligns with Ghana's overall goals of improving food security, increasing farm productivity, and reducing rural poverty and as well as contributing to the achievement of several Sustainable Development Goals (sdgs), such as SDG 1 (No poverty), SGD 2 (Zero hunger), and SGD 12 (Responsible Consumption and Production).

In essence, this study presents a current and relevant method to improving small-scale agriculture in Navrongo by utilizing an advanced multi-objective decision-making model that is scalable and adaptable to other locations facing similar issues.

1.5 Limitations of the study

1. The study considered farming constraints such as land area, labour cost, returns, capital and labour requirement and did not include other inputs like fertilizers application, soil type and water requirement.
2. Few farmers in the area took proper records for their farming activities.
3. Due to the number of decision variables considered in the models and the solution methods employed, optimal solutions are provided in numerical form other than graphical form.

1.6 Organization of the study

This research work is organized into five chapters. Chapter one consists of background to the study, statement of the problem, objectives, significance of the study, limitation and organization of the study. Chapter two presents the literature review. Chapter three introduces the research methods. Chapter four presents the results and discussion. Chapter five consists of the summary, conclusions and recommendations.

1.7 Definition of terms

- **Mathematical Optimization**: a mathematical optimization problem is one in which some real-valued function is either maximized or minimized relative over a set of constraints (Boyd & Vandenberghe, 2004).

- **Objective Function**: is the real-valued function whose value is to be minimized or maximized (Nocedal & Wright, 2006).
- **Decision Variable**: the decision variables in an optimization problem are those variables whose values can vary over the feasible set of alternatives in order to either increase or decrease the value of the objective function (Hillier & Lieberman, 2014).
- **Feasible Region**: the feasible region for an optimization problem is the set of alternatives for the decision variables over which the objective function is to be optimized (Bazaraa et al., 2010).
- **Optimal Solution**: the optimal solution to an optimization problem is given by the values of the decision variables that attain the maximum or minimum value of the objective function over the feasible region (Luenberger & Ye, 2016).
- **Optimal Value**: In an optimization problem where the objective function is to be maximized or minimized, the optimal value is the least upper bound or greatest lower bound of the objective function values respectively over the entire feasible region (Bertsekas, 2016).
- **Linear Program**: a linear program is an optimization problem in finitely many variables having a linear objective function and constraint region determined by a finite number linear equality and/or inequality constraint (Dantzig & Thapa, 1997).
- **Linear Programming** is the study of linear programs: modeling, formulation, algorithms and analysis (Chvatal, 1983).

- **Small-scale farming** refers to agricultural practices where farmers operate on relatively small plots of land, often using traditional methods to produce food primarily for local consumption. the FAO, in a broad definition, considers lands around the world that are smaller than 2 (ha) as small-scale farms (FAO, 2014).

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This chapter presents a review of research on multi-objective optimization. It highlights the diverse study findings published by multiple authors. The review concentrates on an optimization as a tool for solving universal problem, basic concepts of multi-objective optimization, solution of multi-objective optimization and as well as application of optimization in farming. The review points out some knowledge gaps and explains how this thesis will fill them.

2.1 Optimization as a tool for solving universal problems

The domain of optimization pertains to the examination of the maximization and minimization of mathematical functions. Frequently, the parameters of these mathematical functions, which may include known or unknown variables, are governed by ancillary conditions or constraints. Due to its significance, optimization is utilized across multiple disciplines, including applied sciences, engineering, economics, finance, medicine, and statistics. Optimization occupies a pivotal role in both practical applications and the scientific community at large. Its inception dates to the eighteenth century, attributed to the renowned Swiss mathematician and physicist Leonhard Euler (1707-1783), who famously asserted that "nothing at all takes place in the Universe in which some rule of maximum or minimum does not appear." The field of optimization is also referred to as mathematical programming (Beale, 1968).

According to Lewis (2008), optimization originated from George Dantzig, who was a member of the United State Air Force, developed the Simplex method for solving constraint optimization problems in 1947. The objective of the algorithm was to establish an effective methodology for addressing programming challenges characterized by objectives and constraints of a linear nature. Subsequently, scholars from diverse disciplines including mathematics, Operations Research, Economics, and others have advanced the theoretical framework underpinning linear programming and have investigated its various applications.

In contemporary society, optimization encompasses a broad spectrum of techniques derived from Operations Research, Artificial Intelligence, and Computer Science, and is employed to enhance business operations. The term "Programming" in the context of Mathematical Programming differs significantly from that of Computer Programming. In this regard, it signifies the act of planning and organizing, whereas in the realm of computer programming, it refers to the formulation of instructions for executing computations. Although aptitude in one suggests aptitude in the other, training in the one kind of programming has very little direct relevance to the other (Konno and Yamazaki, 1991).

Radhika and Chaparala (2018) provided a foundational overview of optimization techniques, emphasizing the significance of evolutionary metaheuristic approaches in addressing Multi-objective Optimization Problems (MOOP). Their research elucidates the effectiveness of diverse algorithms, including genetic algorithms and particle swarm optimization, in practical applications engineering contexts. They articulate that optimization not only aims to yield effective solutions but also seeks to reduce computational complexity and time, particularly in multi-criterion scenarios where multiple objectives must be balanced. This article sets the stage for understanding optimization as a crucial mechanism in engineering and applied sciences.

Laufer et al. (2023) shift the focus from technical methodologies to the sociotechnical dimensions of optimization. They argue that optimization should be viewed through a lens that recognizes the normative commitments embedded within its processes. Their analysis proposes a systematic pipeline for examining the assumptions and choices involved in optimization, thereby framing it as a method for navigating complex logistical challenges. This perspective highlights the interdependent relationship between optimisation and machine learning, demonstrating how optimisation underpins the development of predictive models. The authors emphasized the ethical dimensions of optimisation, suggesting that the operational efficiency sought through optimisation often carries implicit normative implications that warrant careful examination.

In a more recent contribution, Carissimo and Korecki (2024) studied the limitations of optimization, particularly in the context of modeling complex systems. They argued that while optimization can yield models that simplify and clarify system dynamics, it is fallacious to derive prescriptive conclusions about what ought to be from these models. Their exploration of optimization as a potential form of quantitative ethics raises critical questions about the values underpinning optimization processes. They contend that optimization can mislead stakeholders into adopting a false sense of security regarding the outcomes it produces. Furthermore, the authors draw connections between optimization and ethical frameworks, particularly utilitarianism, to illustrate the ethical quandaries that arise in computational contexts. This critical examination of optimization's boundaries invites a broader discourse on its role and responsibilities in both technological and ethical spheres.

2.2 Basic concepts of multi-objective optimization

Multi-objective programming is an area of multiple criteria decision making that is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously with some set of constraints (Kaisa, 1999). Multi-objective optimization or programming is applied in many real-world problems including problems in the fields of engineering, mining, medicine and finance. In multi-objective optimization deals with multiple conflicting objectives whereby improving one objective will reduce the value of the others, leading to trade-off between solutions. It also assumed that no single solution will optimize all objectives simultaneously because this would be a trivial case (Armand & Malivert, 1993).

Multi-objective optimization's primary goal is to help a decision maker select a preferred option from a range of trade-offs. Because certain solutions could be rejected at each step, it is not required to come up with every solution in this situation when the decision maker is participating in the process. Nevertheless, there are precise, non-interactive techniques that create the complete set of solutions without involving the decision maker. According to Ruzika and Wiecek (2005), there are three forms of multi-objective programming, which includes linear (MOLP), integer (MOIP) and mixed integer (MOMIP), which have continuous, discrete and both continuous and discrete solutions, respectively.

Bhadani et al. (2019) provided a foundational understanding of optimization schemes, demonstrating the effectiveness of MOO through a comparison with Multi-Disciplinary Optimization (MDO). Their study highlights the nature of MOO problems as typically non-linear and complex, necessitating sophisticated mathematical approaches to derive optimal solutions. The authors emphasize that MOO can generate valuable insights for operations in mineral processing, suggesting that similar methodologies may apply to agricultural contexts, where conflicting objectives such as yield maximization and environmental sustainability must be reconciled.

Todman et al. (2019) continued building on the already laid foundation by exploring the utility of multi-objective algorithms in agricultural landscapes. They argue that these algorithms can effectively assess the effects of land management and use techniques across various objectives, thereby revealing trade-offs that might otherwise be overlooked. The complexity of agricultural systems, characterized by heterogeneous landscapes and multiple stakeholders' interests, poses significant challenges for the interpretation of MOO results. However, the authors posit that understanding these trade-offs is essential for making informed decisions that enhance both economic and environmental outcomes.

Garcia and Alamanos (2022) further contributed to this discourse by examining integrated modeling approaches that address sustainable agric-economic growth and environmental improvement. Their work underscores the necessity of taking into account several goals within a unified framework, such as reducing resource consumption, maximizing farmer welfare, and regulating pollution emissions. They contend that combining goal, non-linear, and linear programming optimization methods can yield comprehensive solutions that are particularly relevant in addressing the complexities of agricultural practices. This holistic perspective is crucial for regions like Ghana, where the interplay between agricultural productivity and environmental stewardship is paramount.

However, taking into account a single objective function is used in single-objective optimization, which often yields a single solution known as the optimal solution. A multi-objective optimization task, on the other hand, takes into account multiple competing goals at once. Pareto optimum solutions, also known as non-dominated solutions, are a group of options with varying trade-offs rather than a single ideal answer in such a situation. Even though there are several Pareto optimal solutions, only one of them needs to be selected. Therefore, there are at least two equally significant tasks in multi-objective optimization as opposed to single-objective optimization problems: a decision-making task for selecting the single most preferred solution and an optimization task for locating Pareto optimal solutions using a computer-based procedure. The latter is based on the preference of the decision maker (Jurgen and Kaisa, 2008).

There are three types of multi-objective optimization problems: mixed, linear, and integer. Therefore, in order to provide the foundation for applying multi-objective optimization to solve real-life situations, it is required to comprehend the fundamental ideas of linear and integer programming.

The need to address difficult planning issues during times of war led to the initial development of linear programming (LP) in the 1940s. As numerous sectors discovered the beneficial applications of linear programming in the post-war era, its development advanced quickly. This field is said to have been founded by George B. Dantzig, who created the simplex technique in 1947, and John Von Neumann, who introduced the theory of duality in the same year. Linear programming (LP) problems are optimization problems in which the objective function and all the constraints are linear (Dantzig, 1976).

Also, it has been demonstrated that linear programming is a very effective tool for both modelling real-world issues and as a generally accepted mathematical theory. According to Hillier and Lieberman (2010), linear Programming is a mathematical optimization problem is one in which some function is either maximized or minimized relative to a given set of alternatives. The collection of options is referred to as the feasible region or constraint region, and the function that needs to be minimized or maximized is known as the objective function. One of the key methods for resolving optimization issues is linear programming. It has so many applications in real life; among which are agriculture, energy, construction, manufacturing, health and financial portfolios (Bazaraa *et al*., 2005).

The standard LP form is important for the application of solution algorithms, since the algorithms work only with equality conditions (Lewis, 2008).

McCarl and Spreen (2013) indicated that there are seven (7) main assumptions to consider when building a linear programming problem model. The formulation's suitability is the subject of the first three assumptions, but the model's mathematical relationships are the focus of the final four.

Conversely, a mathematical optimization problem where some or all of the variables are limited to being integers is known as an integer programming issue. When the objective function and constraints (apart from integer constraints) are linear, it is most frequently referred to as integer linear programming (ILP). It is said to be a mixed integer program when some, but not all variables are restricted to be integer and also called pure integer program when all decision variables must be integers (Chen *et al.,* 2010; Hillier and Lieberman*,* 2010).

Multi-objective optimization problems can be solved using a variety of techniques divided into two categories as classical (deterministic) and heuristic (meta-heuristic or non-deterministic) methods but in this study, we will consider some classical methods only.

2.3 Classical methods

The classical approaches that are frequently utilized encompass the Weighted Sum Method, the Epsilon Constraint Method (ε-Constraint Method), the Weighted Metric Method, and the Goal Programming Method. As noted by Giuseppe (2008), classical methodologies can be categorized into four principal types: non-preference methods, posteriori methods, a priori methods, and interactive methods (progressive preference). Interactive methods systematically incorporate preference information throughout the optimization process. According to Cohon and Marks (1975), these techniques normally operate in three stages which are finding a non-dominated solution, getting the reaction of the decision maker regarding this non-dominated solution and modify the preferences of the objectives accordingly and finally repeating the two previous steps until the decision maker is satisfied or no further improvement is possible.

2.3.1 Weighted sum method (WSM)

The weighted sum method represents one of the most effective and straightforward techniques for addressing multi-objective optimization challenges. This methodology is likely the most prevalent due to its inherent simplicity. By employing the weighted sum method, the multi-objective optimization problem is reformulated into a singular objective optimization framework. The weighted sum method assigns weights to each of the objective functions, aggregating them into a single objective function and parametrically varying the weights to generate the non-dominated set (Konak and Coit, 2006).

In a pivotal study by (Krylovas et al., 2017), the authors conducted an empirical analysis of WSM alongside other MCDM methods such as AHP and TOPSIS, specifically in the context of sustainable housing affordability and the selection of wind turbine support structures. This comparative analysis underscores WSM's practicality in real-world applications, demonstrating its effectiveness when juxtaposed with other methodologies.

Mokhtar et al. (2017) further explored the utility of WSM, by examining its role in selecting the best demand-side management (DSM) options. Their research emphasizes the importance of MCDM in optimizing energy usage and peak demand reduction. The authors integrated WSM with the Analytic Hierarchy Process (AHP) to establish relative importance among criteria, thus enhancing the decision-making process. This combination not only illustrates WSM's adaptability but also its reliance on subjective weighting methods, which can be critical in complex decision environments.

In a more recent contribution, (Sorooshian & Parsia, 2019) delved into the limitations of WSM, particularly its challenges in processing information from diverse sources. Their study highlights the method's accessibility for practitioners with limited mathematical expertise while also pointing out the significant constraint of WSM in situations where decision-makers may lack comprehensive knowledge of specific criteria. This critical evaluation serves to inform potential enhancements and adaptations of the WSM, aiming to address its shortcomings while maintaining its applicability in various decision-making contexts.

2.3.2 Epsilon constraint method (ε -Constraint Method)

In addition to the weighted sum or scalarization methodology, an alternative solution technique for addressing multi-objective optimization challenges is the ε-constraint method (Chankong and Haimes, 1983). In this context, the decision maker selects one objective from the n available to be minimized, while the other objectives are subjected to constraints that require them to be less than or equal to specified target values. The ε-constraint method methodically alters the search space or feasible criterion set by adjusting the values of the upper bounds corresponding to the criteria and subsequently solving the resulting scalar optimization problems. Through this process, a substantial portion of the Pareto optimal set can be generated. The decision maker has the ability to articulate relative preferences for each criterion by judiciously selecting the values of the upper bounds. However, these selections must be feasible to guarantee the attainment of Pareto optimal solutions. However, there are some advantages and disadvantages associated with the constraint method (Giuseppe, 2008). The foundational work by (Fan et al., 2017) establishes a comprehensive framework for understanding Constraint Multi-objective problems (CMOPs), highlighting the critical role of converting equality constraints into inequality constraints through the introduction of a small positive number, epsilon. This method makes it easier to identify Pareto optimal solutions, which are crucial for multi-objective optimization, in addition to simplifying the representation of constraints. The authors emphasize the importance of balancing objectives and constraints, introducing both static and dynamic penalty function approaches to manage constraint violations effectively. Vaz et al. (2020) built on the earlier works by delving into the practical application of constraint handling techniques (CHTs) within the context of the Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D). Their exploration reveals the challenges associated with finding optimal solutions in complex real-world scenarios, particularly under budget constraints for evaluations. The introduction of the Three Stage Penalty technique represents a significant advancement, demonstrating competitive performance in terms of hypervolume values. This work underscores the need for tailored CHTs that can adapt to the specific characteristics of the problem at hand, reinforcing the notion that there isn't a solution that works for everyone in multi-objective optimization. Furthering the discourse, (Rahimi et al., 2022) provided a thorough review of constraint handling techniques across both single-objective and multi-objective evolutionary algorithms. Their findings indicate a notable disparity in the attention given to multi-objective optimization compared to single-objective cases. The authors compile a comprehensive analysis of various CHTs, identifying genetic algorithms, differential evolutionary algorithms, and particle swarm intelligence as particularly promising for tackling the complexities of multi-objective optimization. This review not only highlights the advancements in the field but also calls for more focused research on CHTs specific to multi-objective scenarios, suggesting that the integration of evolutionary algorithms with effective CHTs remains a crucial area for future exploration.

2.3.3 Weighted metric method (WMM)

This also one of the often-used classical method for solving multi-objective optimization problems. Under this the idea is instead of using a weighted sum of the objectives, we rather consider other ways of combining multiple objectives using metrics (Giuseppe, 2008).

2.3.4 Goal programming method (GPM)

With this approach, the researcher must define objectives for every aim they hope to accomplish. Goal programming's primary concept is to identify solutions for one or more objective functions that achieve a predetermined target. So, if no solution reaches the predefined goals in all the objective functions, the task is to find solutions that minimize deviations from those goals (Deb, 2001). The goal programming or attainment technique involves a set of design goals, $F^* = \{F^*_1, F^*_2, ..., F^*_m\}$, which is associated with a set of objectives $F(x) = \{F_1(x), F_2(x), ..., F_m(x)\}$. The problem formulation also allows the objectives to be underachieved or overachieved, enabling the designer to be relatively imprecise about initial design goals (Gembicki, 1974).

2.4 Evolutionary Algorithms and Constrained Methods

The development of constrained multiobjective evolutionary algorithms (CMOEAs) is a pivotal advancement in evolutionary methods for MOO. Mirjalili (2018) emphasized the systematic review of CMOEAs, illustrating how these algorithms have evolved to handle constraints effectively, a significant challenge in MOO. Liang et al. (2023) further built on this by providing a comprehensive survey of CMOEAs, categorizing various algorithms and analyzing their performance. Such insights are essential for understanding the landscape of constraint handling in evolutionary strategies, particularly in addressing constrained multiobjective optimization problems (CMOPs).

The benchmarking of CMOEAs against established methods like NSGA-II and SPEA2 provides a comparative perspective on their effectiveness (Mirjalili, 2018; Liang et al., 2023). These studies underscore the necessity of developing robust constraint handling techniques, which remain a critical area of research within MOO.

2.5 Metaheuristic Methods

The integration of metaheuristic algorithms with machine learning techniques has shown promise in various applications, including concrete mixture optimization (Zhang et al., 2020) and building energy performance (Chegari et al., 2021). These applications highlight the versatility of metaheuristic methods in addressing complex optimization tasks across different domains. For instance, the optimization of concrete mixtures using machine learning and metaheuristic algorithms reflects a practical implementation of MOO in the construction industry, illustrating the real-world applicability of these methods.

In the context of high-performance computing, Wang et al. (2020) showcased the efficacy of evolutionary multi-objective optimization algorithms for cyber-physical social systems. This highlights the increasing relevance of MOO in emerging technological domains, where computational efficiency and optimization are paramount.

2.6 Other techniques in multi-objective optimization

2.6.1 Multi-Criteria Decision Analysis (MCDA)

MCDA encompasses various methodologies designed to aid decision-makers in evaluating alternatives based on multiple criteria. The extensive literature in this field highlights several key approaches and their applications across different domains. For instance, the Analytic Hierarchy Process (AHP) established by Saaty (1980) remains one of the most widely adopted methods, facilitating the structuring of complex problems into a hierarchical format and allowing for pairwise comparisons among criteria (Marttunen et al., 2017). Recent advancements in MCDA have integrated fuzzy logic to address the inherent uncertainty in human judgments. Zadeh's (1965) introduction of fuzzy set theory has been instrumental in refining decision-making processes, especially in contexts where the vagueness of criteria is prominent (Sahoo & Goswami, 2023). Furthermore, hybrid methodologies that combine different MCDA techniques have emerged, enhancing the robustness and adaptability of decision-making frameworks (Marttunen et al., 2017).

The application of MCDA spans various fields, including healthcare, environmental management, and engineering. For instance, Kavzoglu et al. (2014) utilized GIS-based MCDA for landslide susceptibility mapping, demonstrating the practical implications of these decision-making frameworks. Additionally, the healthcare sector has seen significant contributions from MCDA, with studies assessing the value of interventions and incorporating uncertainty into decision-making processes (Marsh et al., 2014; Siksnelyte-Butkiene et al., 2020).

Despite the wealth of methodologies and applications, there remain substantial gaps in the literature, particularly concerning the integration of big data analytics and machine learning techniques into MCDA. Future research should focus on exploring these intersections to enhance decision-making processes and address the complexities of contemporary challenges.

2.6.2 Pareto Optimality

The concept of Pareto optimality is central to multi-objective optimization problems (MOPs), where the goal is to find solutions that cannot be improved in one objective without degrading another. This principle, introduced by Vilfredo Pareto, has garnered significant attention in recent decades, particularly within the evolutionary computation community (Tian et al., 2020). The Non-dominated Sorting Genetic Algorithm II (NSGA-II) developed by Deb et al. (2002) serves as a benchmark for evaluating new algorithms designed to navigate the complexities of Pareto optimal solutions.

The literature indicates a growing interest in addressing sparse Pareto optimal solutions, where many decision variables may be zero. Zhang and Li (2007) proposed algorithms specifically targeting these sparse solutions, indicating that the optimization community recognizes the unique challenges presented by high-dimensional spaces. Recent advancements in evolutionary algorithms have introduced novel population initialization strategies and genetic operators to enhance the identification of Pareto optimal solutions (Tian et al., 2020).

In the context of chemical engineering, the literature has explored various methods for selecting optimal solutions from the Pareto-optimal front. Wang et al. (2017) highlighted that the selection process is often influenced by specific application contexts, yet the methods for this crucial selection phase remain underexplored. Future research should focus on refining these selection methods and their application across different industries, particularly in environments characterized by multiple conflicting objectives.

2.6.3 Dominance

Dominance, closely related to Pareto optimality, is a vital concept in the evaluation of alternatives in multi-objective optimization. A solution is deemed dominant if it outperforms another solution in at least one objective without being inferior in any other. This principle serves as a foundational criterion for assessing the quality of solutions within the Pareto front. The integration of dominance in decision-making frameworks has facilitated the identification of superior solutions in various applications, including feature selection in machine learning and optimization in engineering contexts. However, the literature on dominance remains sparse, particularly regarding the development of methodologies that effectively incorporate dominance into broader decision-making frameworks. The exploration of dominance-related concepts in sparse optimization problems has also gained traction, as researchers seek to understand how to effectively utilize dominance in scenarios where solutions may be sparse (Conitzer et al., 2016). Future research directions should focus on developing robust methodologies that enhance the understanding and application of dominance in multi-objective optimization contexts.

2.7 Application of optimization in farming

Otieno and Adeyemo (2011) investigated the multi-objective cropping pattern in South Africa's Vaalharts irrigation system. The model was created with three primary presumptions: to reduce irrigation water use, maximize total agricultural output, and maximize the net benefit in monetary terms that would result from planting four distinct crops. after the multi-objective model of this study was solved using the multiobjective differential evolution technique that was created. The averages of the total net benefit, total agricultural output, total irrigation water, and total planted area were found to have improved, coming in at ZAR 882,890.63, 3439518.75 tons, 702522.50m3, and 661444.06m2, respectively. Also, Van Wijk et al., (2012) emphasizes the critical role of multi-objective optimization models in enhancing sustainability under climatic uncertainty. Their research illustrates how integrated crop-livestock simulation models can facilitate scenario analysis and impact assessment, providing essential tools for strategic management in small-scale farming systems. This article sets the stage for understanding the necessity of adaptive strategies in farm planning amidst changing environmental conditions.

Zheng et al. (2013) studied multi-objective firework optimization for variable-rate fertilization in oil crop production for East China. By simulating a fireworks explosion, the hybrid multi-objective fireworks optimization algorithm (MOFOA) produced a set of solutions to the Pareto optimal front. The study presented a multi-objective optimization problem model that considered not only crop yield and quality but also energy consumption and environmental effects for oil crop fertilization. The efficiency and applicability of the algorithm were proven by their experimental trials and actual uses in the production of oil crops in eastern China. In an environment where water is scarce, the model was determined to be a good substitute for farmers to acquire the best crop planning. Similarly, Otoo et al. (2015) studied the optimal selection of crops of small-scale farms in the Fanteakwa District of Ghana, using linear programming and reveals that, the LP model saved 0.2% and 0.6% of the available capital and labour requirement respectively and also saw an increase of 16.25% in the net returns. Igwe and Onyenweaku (2013) applied linear programming technique to food crops and livestock enterprises planning in Aba agricultural zone of Abia State, Nigeria and realized an increment of 61.35% in the existing margin. The study also suggested that for the farmer to improve gross margin they should consider crops, poultry and fish farming. Adekanmbi and Olugbara (2015) used multi-objective optimization for crop-mix planning using the general differential evolution algorithm. Using data from the South African Grain Information Service, this study's limited multi-objective optimization model demonstrated that the generalized differential evolution 3 algorithm could effectively solve optimal mixed-cropping planning problems.

Majeke (2013) conducted a study at Bindura in Zimbabwe where they applied linear programming as a tool to help farmers in the selection of combination of farm activities which was given in a feasible set of fixed farm constraints and was to maximize income whiles achieving other goals such as food security. It was realized that, there was 44.65% increase in the gross income utilizing the model of linear programming as compared to the normal traditional methods. Mugabe (2014) used linear programming to estimate optimal land use allocation among small holder (A1) farmer households in Zimbabwe, their study revealed that, A1 households are insecure with respect to their land holdings and therefore cannot use their land as farmers to access credit as collateral security. Also, there was an optimal allocation of 16.09, 169.92 and 25.24 hectares of land for maize, soyabeans and sugar beans respectively. Similarly, Sofi (2015) worked on decision making in agriculture using linear programming approach to determine the optimum land allocation for five crops and observed that, the proposed LP model was very appropriate for finding the optimal land allocation to the major crops at 2752.56 acres and achieved a maximum profit of Rs. 1376.00.

Zhai et al. (2018) introduced a mission planning approach that further refines the application of multi-objective optimization in precision farming systems. They delineate the dual objectives of expected benefits and costs, highlighting sub-objectives that include efficiency and energy consumption. Their findings underscore the importance of balancing these competing objectives to optimize agricultural missions effectively. This perspective is particularly relevant for small-scale farmers who must navigate the constraints of limited resources while striving for productivity and sustainability. Similarly, Nagy et al. (2018) worked on application of the multiple objective programming in the optimization of production structure of an agricultural holding and their work revealed some opportunities and the importance of Multiple Objective Linear Programming methods which were examined that is the applicability of goal programming and that of MOLP through an example of an agricultural enterprise.

Radhika and Chaparala (2018) provided a comprehensive overview of evolutionary metaheuristic techniques, emphasizing their role in solving Multi Objective Optimization Problems (MOOP). They pointed out that these evolutionary approaches, including genetic algorithms and particle swarm optimization, have gained traction in various engineering applications, including agriculture and then underscored the importance of optimizing computational time and complexity, which has led to the emergence of heuristic search algorithms that are particularly useful in addressing the intricate challenges of agricultural optimization. Todman et al. (2019) expanded the discussion by exploring how multi-objective optimization can be employed to identify potential future agricultural landscapes. Their work revealed the complexity of trade-offs inherent in agricultural management decisions and the need for stakeholder engagement to navigate these challenges. The authors advocated for the use of optimization algorithms to visualize and analyze the effects of different management strategies on various objectives, thus fostering informed discussions among stakeholders. This participatory approach is essential for collectively identifying priorities and managing agricultural systems to meet diverse objectives.

Yang et al. (2020) also presented an algorithm to solve fuzzy multi-objective linear fractional programming (FMOLFP) problems through an approach based on superiority and inferiority measures method (SIMM). The results showed that the acreage of cotton should be 20,669.6 ha, while the acreage of winter wheat and summer corn should be 38,386.4 ha. Each fuzzy goal specified for the fractional objectives and some of the constraints in their model had fuzzy values. The ratio of grain planted area to cotton planted area is irrational due to the current high risk of cotton agriculture. For the government to increase farmers' motivation to plant cotton and maintain the long-term growth of the cotton market, better policy assistance is required.

Jian et al. (2021) developed models to reduce fertilizer use and enhance net crop benefit as multi-objective optimization functions, and suggested a hybrid CSA-PSO optimization technique for resolving multi-objective issues by fusing the CSA with optimization of particle swarms (PSO). The suggested algorithm's performance was assessed using benchmark functions from CEC 2009. The findings demonstrated the viability and efficacy of the suggested method, which was used to optimize crop patterns in the Telangana state of India. Begam et al. (2023), their research showed the effectiveness of multi-objective particle swarm optimization in maximizing net returns while minimizing water usage, showcasing its application in resource-scarce regions. Similarly, Paez et al. (2023) employed a multi-objective optimization framework for the strategic planning of preventive measures aimed at drought management, highlighting the inherent trade-offs between agricultural and hydrological drought management. Their methodology illuminates the intricacies involved in resource allocation amidst the challenges posed by climate variability.

Osika et al. (2023) conducted a survey on decision-support methods for multi-objective optimization, focusing on the exploration of solutions beyond the Pareto front. This work is particularly relevant as it addresses the challenges of visualizing and interpreting the solutions generated by multi-objective optimization algorithms. They presented an emerging research directions, including the need for interactivity and explainability in decision-support tools, which could enhance the applicability of multi-objective optimization in real-world agricultural scenarios. Their insights suggest a growing recognition of the ethical considerations and complexities inherent in optimizing agricultural systems.

Fathollahi-Fard et al. (2023) used a fuzzy logic multi-objective optimization model to optimize profit while minimizing greenhouse gas emissions and waste in sustainable harvest planning. Also, Ghersa et al. (2024), considered the AgrOptim framework which combines crop simulation with genetic algorithms to optimize economic and biophysical indicators, revealing trade-offs between economic returns and environmental impacts. Similarly, Randall et al. (2024), also emphasized the need for optimization models to integrate climate projections, addressing the increasing tension for water resources and the complexities of crop planning under climate change.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

This chapter discusses the general multi-objective optimization problem; the data collection processes and the formulated multi-objective models which seek to maximize net returns and minimize labour cost simultaneously, as well as the solution methods models.

3.1 Multi-objective programming

The multi-objective optimization problem can be formulated mathematically as follows similar to (Kaisa, 1999);

$$\text{Minimize}\left[f_1(x), f_2(x), \dots, f_k(x)\right]^T$$

Subject to; $g_j(x) \leq 0 \qquad j = 1, 2, \dots, k$

$$h_j(x) = 0 \qquad j = k+1, \dots, p \qquad (3.1)$$

Or the problem can be simply posed as;

$$\text{Minimize}\left\{f(x) : x \in S\right\} \qquad (3.2)$$

Where the integer $k \geq 2$ is the number of objectives and S is the feasible set of decision alternatives and is defined by $S = \left\{x : g_j(x) \leq 0, j = 1, 2, \dots, k,\ h_j(x) = 0\ \ j = k+1, \dots, p\right\}$.

If some objective function is to be maximized, it is equivalent to minimize its negative.

3.1.1 Steps in formulating a multi-objective programming model

For multi-objective programming to be used and used effectively, a realistic model that truly reflects the decision-making objectives within the constraints that must be met must be developed. According to Ehrgott, 2005 Creating a multi-objective programming paradigm involves the following fundamental steps:

Step 1: Identification of the decision variables.

The decision variables (parameters) having a bearing on the decision at hand shall first be identified.

Step 2: Identification of the constraints.

All the constraints in the given problem which restrict the operation of a decision maker or firm at any given point of time must be identified in this stage. Further, these constraints should be broken down as linear or non-linear functions in terms of the pre-defined decision variables.

Step 3: Identification of the objectives.

The objectives which are required to be optimized (i.e., maximized or minimized) must be clearly identified and expressed in terms of the pre-defined decision variables.

Step 4: Write the formulation.

a) Write the formulation table taking into account step 1- 3.

b) Write the algebraic multi-objective programming formulation.

3.2 Formulation of the problem

The model seeks to maximize returns and minimize labour cost subject to available land area for cultivation, the capital requirement for all the crops and the available labour for all the crops. Due to these conflicting objectives, the crop production model is formulated using the multi-objective optimization approach.

3.2.1. Model parameters

The following are the multi-objective programming model's parameters:

R: The total net returns from all the crops in (GH¢).

L: The total cost of labour in (GH¢).

n: The number of crops.

N_i: The net return from ith crop per acre in (GH¢).

C_i: The cost of labour required for each ith crop per acre in (GH¢).

X_i: The area under ith crop in acres.

Z_i: The capital requirement for ith crop per acre in (GH¢).

L_i: The labour requirement for the ith crops per acre.

T_{Area}: Total land area for cultivation of crops in acres.

$T_{Capital}$: The total capital requirement for all the crops.

T_{Labour}: The total labour requirement for all the crops.

3.2.2 Decision variables of the model

The developed model's decision variables are the land areas used for the production of the various crops.

3.2.3 Objective functions of the model

3.2.3.1 Objective function 1: Returns maximization

The principle on which the model is based is the principle of profit maximization, where the farmer is to choose a production plan that is likely to maximize returns. The total returns of an individual farmer depend on the returns he gets from the number of crops he grows. This is expressed as follow:

$$\text{Maximize } R(X) = \sum_{i=1}^{n} N_i X_i,\ i = 1, 2, \ldots n \tag{3.3}$$

3.2.3.2 Objective function 2: Labour cost minimization

Again, given the choice of the farmer to maximize the returns or crop production and also meeting the food demand of the people, cultivation of profitable crops is dependent on the amount invested in labour. Crop production or returns maximization will therefore require minimizing the cost of labour. This is expressed as follows:

$$\text{Minimize } L(X) = \sum_{i=1}^{n} C_i X_i, i = 1, 2, \ldots n \tag{3.4}$$

3.2.4 Constraints and resulting model

3.2.4.1 Constraint 1: Land usage

The total land used for the given types of crops must not be greater than the total available land area for cultivation. This is expressed as follows:

$$\sum_{i=1}^{n} X_i \le T_{Area} \tag{3.5}$$

3.2.4.2 Constraint 2: Capital

The total investment in the crop production must not be greater than the working capital. This is expressed as follows:

$$\sum_{i=1}^{n} Z_i X_i \le T_{capital} \tag{3.6}$$

3.2.4.3 Constraint 3: Labour

The total number of labour for all crops must not be greater than the total number of labour required. This is expressed as follows:

$$\sum_{i=1}^{n} L_i X_i \le T_{Labour} \tag{3.7}$$

The proposed multi-objective programming model for farm planning is therefore given as follows:

Maximize $R(X) = \sum_{i=1}^{n} N_i X_i ,\ i = 1, 2, \ldots n$

Minimize $L(X) = \sum_{i=1}^{n} C_i X_i , i = 1, 2, \ldots n$

Subject to; (3.8)

$$\sum_{i=1}^{n} X_i \le T_{Area}$$

$$\sum_{i=1}^{n} Z_i X_i \le T_{Capital}$$

$$\sum_{i=1}^{n} L_i X_i \le T_{Labour}$$

$$X_i \ge 0 \; for \, i = 1, 2, \ldots n$$

3.3 Solution methods

There are several solution methods that are used to find the solution of a multi-objective optimization problem namely weighted sum, epsilon constraint, goal programming and compromise programming. This study will adopt the weighted sum and the epsilon-constraint methods because these techniques provide flexible, scalable, and efficient ways to solve complex multi-objective optimization problems. Their ability to balance trade-offs, consider real-world constraints, and apply to a wide range of disciplines ensures they remain critical tools for advancing research and solving practical, multifaceted problems.

3.3.1 Resulting weighted sum model

The weighted sum approach takes the decision-maker's preferences into account while solving optimization models. This is expressed mathematically as:

$$\text{Minimize } \sum_{i=1}^{k} w_i f_i(x) \tag{3.9}$$

$$\text{Subject to; } \quad x \in S$$

$$w_i > 0, \forall\, i = 1, \ldots, k \text{ and } \sum_{i=1}^{n} w_i = 1$$

Where w_i is the weight of the ith objective function which indicates the preference of the decision maker and S is the feasible decision set. The weighted sum method will give an approximation of the non-dominated set, because the non-dominated set could have infinite solutions so the cluster technique will be applied to minimize the non-dominated set to a smaller set, which will give some flexibility to the decision maker (Kim and De-Weck, 2006).

In this study the weights of the model are taken according to the preferences of the farmers in the selected areas. Also, one model which comprises the two objectives and the available constraints is considered. It is presented as follows;

3.3.1.1 Model 1: Maximizing returns and Minimizing labour cost using the weighted sum

This model seeks to maximize returns and minimize cost of labour with the weights preferred by the individual farmers. The model is expressed as follows:

$$\text{Maximize } F_3(X) = w_1 \sum_{i=1}^{n} N_i X_i - w_2 \sum_{i=1}^{n} C_i X_i \qquad (3.10)$$

Subject to;

$$\sum_{i=1}^{n} X_i \leq T_{Area}$$

$$\sum_{i=1}^{n} Z_i X_i \leq T_{Capital}$$

$$\sum_{i=1}^{n} L_i X_i \leq T_{Labour}$$

$$X_i \geq 0 \; for \, i = 1, 2, \ldots n$$

$$w_1 + w_2 = 1 \quad and \; w_1, w_2 \geq 0$$

where w_1 and w_2 are the preferred weights for the first and second objective functions respectively

3.3.2 The resulting espilon (ε) constraint models

The epsilon constraint method of solving optimization models consider taking one objective function at a time while the other objective functions are treated as part of the constraints. The problem thus becomes:

$$\text{Minimize } f_i(x) \qquad 1 \le i \le m \qquad (3.11)$$

$$\text{Subject to: } \quad x \in S$$

$$f_j \le U_j, \qquad j = 1, 2, ..., i-1, i+1, ..., m$$

Where $U_j \; \forall j \in \{1, 2, ..., i-1, i+1, ..., m\}$ is the j^{th} criterion upper bound.

In this study, considering the two objectives and taking one at a time while the other objective becomes part of the constraints gives us two different models as follows:

3.3.2.1 Model 2: Maximizing returns using the espilon (ε) constraint method

This model seeks to maximize returns while the labour cost is treated as a constraint where the total cost of labour for all crops must not be greater than the total labour cost. The resulting model is therefore given as follows:

$$\text{Maximize } R(X) = \sum_{i=1}^{n} N_i X_i, \; i = 1, 2, ...n \qquad (3.12)$$

Subject to;

$$\sum_{i=1}^{n} C_i X_i \le T_{labour\,cost}$$

$$\sum_{i=1}^{n} X_i \le T_{Area}$$

$$\sum_{i=1}^{n} Z_i X_i \leq T_{Capital}$$

$$\sum_{i=1}^{n} L_i X_i \leq T_{Labour}$$

$$X_i \geq 0 \; for \, i = 1,2,...n$$

3.3.2.2 Model 3: Minimizing cost of labour using the espilon (ε) constraint method

This model is aimed at minimizing cost of labour while the returns is treated as a constraint where the total returns of all crops must not be greater than the total returns. The model is expressed as follows;

Minimize $L(X) = \sum_{i=1}^{n} C_i X_i, \; i = 1,2,...n$ (3.13)

Subject to;

$$\sum_{i=1}^{n} C_i X_i \leq T_{labour\cos t}$$

$$\sum_{i=1}^{n} X_i \leq T_{Area}$$

$$\sum_{i=1}^{n} Z_i X_i \leq T_{Capital}$$

$$\sum_{i=1}^{n} L_i X_i \leq T_{Labour}$$

$$X_i \geq 0 \; for \, i = 1,2,...n$$

Where $T_{\mathrm{Re}\,turns}$ is the total returns expected from all crops and $T_{Labour\cos t}$ is the total cost of labour requirement for all crops.

3.4 Data collection

The data for this study is mainly primary which was obtained from some selected small-scale farmers from Kologo, Gani and Tono all in the Kesena Nankana East Municipality of the Upper East region of Ghana. These farmers were selected using purposive and convenient sampling based on reason that the major crops grown in the study area are cropped by them for commercial purposes. These sampling techniques were chosen because purposive sampling enabled the researcher to deliberately select specific farms or farmers who are actively involved in decision-making regarding resource allocation, crop selection, and land use, while convenient sampling allowed the researcher to include participants who are readily available and willing to participate within the study's timeframe. The data were mainly from irrigated crops, taking into consideration that most of the farmers in the study area mainly relied on the major irrigation facility that serves the farmers around Tono and its environs. The data consist of: land area used for cultivation, the net returns after production, the cropping patterns, capital and labour requirements. A pilot study was conducted in Gani to test the reliability and validity of the models before the main data collection.

3.5 Data analysis

The data collected was analysed with the help of a mathematical programming software known as Management Scientist v6.0 (Anderson *et al.,* 2004). The management scientist is a mathematical programming software for quantitative analysis and operational management that helps to solve all integer and mixed integer programming problems. Also, this programming software has modules within that help in solving optimization problems like Linear Programming, Transportation Problem, Assignment Problem, Integer Linear Programming, Shortest Route, Minimal Spanning Tree, Program Evaluation and Review Technique and Critical Path Method.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Introduction

This study aimed at using the multi-objective programming techniques to formulate a mathematical programming model that will help farmers to either maximize returns, minimize labour cost or maximize returns and minimize labour cost simultaneously subject to available land area for cultivation, the capital requirement for all the crops and the available labour requirement for all crops. After successfully formulating the models, we are applying them to the collected data to see their efficiency in real-life situations.

Thus, the data, data analysis, computational results, and in-depth discussions of the computational results are presented in this chapter.

4.1 Results for major vegetable farming

The developed model has been applied to some selected farmers at Kologo-Naga and Tono in Navrongo (Upper East Region of Ghana) via face-to-face interview. The data used were collected from two sets of farmers, those who grow the major vegetables and those who grow cereal/ legumes. The first set of farmers were selected based on the major vegetables grown in the study area for commercial purposes. These major vegetables grown are pepper, okro, onion, garden eggs and tomatoes. Table 4.1 gives the five major vegetables, land area (in acres), number of labourers, labour cost (in GH¢), capital (in GH¢) and returns (in GH¢).

Table 4.1: Overview of major vegetables production

S/No.	Vegetables	Land Area (in acres)	Number of labourers	Labour Cost (in GH¢)	Capital (in GH¢)	Returns (in GH¢)
1.	Pepper	6	12	1320	3900	4870
2.	Okro	3	6	520	1100	1550
3.	Onions	2	6	720	1000	1400
4.	Garden eggs	4	8	800	1300	1760
5.	Tomatoes	3	6	400	1200	1980
	TOTAL	**18**	**38**	**3760**	**8500**	**11560**

Source: Field Survey, 2024

Based on the collected data, Table 4.2 which gives the major vegetables with their corresponding land area, number of labourers, labour cost, capital and returns per acre was obtained as follows:

Table 4.2: Overview of major vegetables per acre

S/No.	Vegetables	Land Area (in acres)	Number of labourers per acre	Labour Cost (in GH¢) per acre	Capital (in GH¢) per acre	Returns (in GH¢) per acre
1.	Pepper	1	2	220	650	811.67
2.	Okro	1	2	173.33	366.67	516.67
3.	Onion	1	3	360	500	700
4.	Garden eggs	1	2	200	325	440
5.	Tomatoes	1	2	133.33	400	660

Source: Field Survey, 2024

4.1.1 Maximizing returns and minimizing labour cost using the weighted sum

With the help of a programming software known as Management Scientist (Anderson, et al., 2004), the collected data was analysed. For Model 1, using a scalarisation weight of 0.6 for returns and 0.4 for labour cost. The results are as follows;

Maximize

$$0.6(811.67X_1 + 516.67X_2 + 700X_3 + 400X_4 + 600X_5) - 0.4(220X_1 + 173.33X_2 + 360X_3 + 200X_4 + 133.33X_5)$$

Maximize

$$(487 - 88)X_1 + (310 - 69.33)X_2 + (420 - 144)X_3 + (240 - 80)X_4 + (360 - 53.33)X_5$$

Resulting in:

Maximize $399X_1 + 240.67X_2 + 276X_3 + 160X_4 + 306.67X_5$

Subject to;

$$650X_1 + 366.67X_2 + 500X_3 + 325X_4 + 400X_5 \leq 8500$$

$$X_1 + X_2 + X_3 + X_4 + X_5 \leq 18$$

$$2X_1 + 2X_2 + 3X_3 + 2X_4 + 2X_5 \leq 38$$

$$X_1, X_2, X_3, X_4, X_5 \geq 0$$

where X_1 is land area for pepper production in acres, X_2 is the land area for okro production in acre, X_3 is the land area for onions production in acres, X_4 is the land area for garden eggs production in acres and X_5 is land area under tomatoes production in acres. Table 4.3 gives the optimal solution for model 1.

Table 4.3: Optimal solution for model 1 for vegetable production

Decision Variables	Optimal acres of land for cultivation	Optimal returns (in GH¢)
X_1	4.40	
X_2	0.00	
X_3	2.00	6460.98
X_4	0.00	
X_5	11.60	
Total land in acres	**18.00**	

From Table 4.3, it is seen that the optimal solution is GH¢ 6460.98 and proposes production of 4.40 acres of pepper, 2 acres of onions and 11.60 acres of tomatoes. It was observed that if there is no production of okro and garden eggs, the farmer would still achieve an optimal return of GH6,460.98.

Table 4.4 gives the comparism of the existing and optimal cropping patterns for vegetables (together with their associated returns).

Table 4.4: Comparison of existing and optimal cropping patterns for vegetables (together with their associated returns)

Crops	**Existing cropping Plan Size of farm (in acres)**	**Optimal Cropping plan Size of Farm (in acres)**
Pepper	6.00	4.40
Okro	3.00	0.00
Onion	2.00	2.00
Garden eggs	4.00	0.00
Tomatoes	3.00	11.60
Total acres	18.00	18.00
Total Returns (in GH¢)	**5,432.00**	**6,460.98**

From Table 4.4, it is observed that, for a farmer using the existing cropping pattern make 60% increase in his returns and concurrently decrease his labour cost by 40%, he has to decrease and increase the lands used to grow pepper and tomatoes by 1.60 and 8.60 acres respectively whiles maintaining the same land used in cultivating onion. Also, the difference in returns is $6460.98 - 5432.00 = 1028.98$ which indicates approximately 19 % increase in the existing returns.

4.1.2 Sensitivity analysis on model 1 for vegetables farming

The sensitivity analysis conducted for the weighted sum vegetable model indicates that various coefficients associated with the objective function and the corresponding right-hand-side (RHS) constraint values can be modified within specified limits without undermining the optimal solution.

In particular, the cost coefficients pertaining to decision variables X_1 through X_5 are permitted to fluctuate within established bounds: for instance, the coefficient for X_1 may oscillate between 324.67 and 527.589 whiles the coefficients for X_5 also oscillates between 259.295 and 399.00, whereas X_2, X_3 and X_4 possess no lower limits on their respective coefficients, their upper limits are 314.76, 354.402 and 302.371 respectively.

Moreover, the RHS values of constraints reveal permissible ranges. For instance, the RHS of Constraint 1 is allowed to vary from 7200.00 to 11,700, while Constraint 2 permits a variation range from 13.077 to 19.00. Similarly, Constraint 3 is capable of varying from a lower limit of 36 with no upper limits. This analysis substantiates the robustness of the solution within these specified ranges, thereby providing valuable insights into potential adjustments in resource allocation without jeopardizing the solution for the weighted sum model that maximize returns whiles minimize labour cost.

4.1.3: Maximizing returns using the espilon (ε) constraint method

The formulated model for maximization of returns was used and in solving it, only the objective for returns was considered while the labour cost minimization objective was treated as a constraint together with the other constraints. Based on the collected data, the resulting model is as follows;

Maximize $811.67X_1 + 516.67X_2 + 700X_3 + 400X_4 + 600X_5$

Subject to;

$$650X_1 + 366.67X_2 + 500X_3 + 325X_4 + 400X_5 \leq 8500$$

$$X_1 + X_2 + X_3 + X_4 + X_5 \leq 18$$

$$2X_1 + 2X_2 + 3X_3 + 2X_4 + 2X_5 \leq 38$$

$$220X_1 + 173.33X_2 + 360X_3 + 200X_4 + 133.33X_5 \leq 3760$$

$$X_1, X_2, X_3, X_4, X_5 \geq 0$$

Table 4.5 gives the optimal solution of model 2 for vegetable production.

Table 4.5: Optimal solution for model 2 for vegetables production

Decision Variables	Optimal acres of land for cultivation	Optimal returns (in GH¢)
X_1	5.00	
X_2	0.00	
X_3	0.00	12668.68
X_4	0.00	
X_5	13.00	
Total land in acres	**18.00**	

From Table 4.5, it is seen that the optimal solution is GH¢ 12,668.68 and proposes production of 5 acres of pepper and 13 acres of tomatoes. It again proposes no production of okro, onion and garden eggs.

Table 4.6 gives the comparism of the existing and optimal cropping patterns (together with their associated returns) for vegetables.

Table 4.6: Comparison of existing and optimal cropping patterns (together with their associated returns) for vegetables

Crops	Existing cropping Plan Size of farm (in acres)	Optimal Cropping plan Size of Farm (in acres)
Pepper	6.00	5.00
Okro	3.00	0.00
Onion	2.00	0.00
Garden eggs	4.00	0.00
Tomatoes	3.00	13.00
Total acres	18.00	18.00
Total Returns (in GH¢)	**11,560.00**	**12,668.68**

From Table 4.6, it is observed that, for a vegetable farmer interested in maximizing returns only, he has to decrease the land used to grow pepper by 1 acre and increase that of tomatoes by 10 acres. Also, the difference in net returns is $12668.68 - 11560.00 = 1108.68$ which indicates approximately 10% increase in the existing net returns.

4.1.4 Sensitivity on model 2 for vegetables farming

The sensitivity analysis conducted for the vegetables returns maximization model indicates that various coefficients associated with the objective function and the corresponding right-hand-side (RHS) constraint values can be modified within specified limits without undermining the optimal solution.

In particular, the return coefficients pertaining to decision variables X_1 through X_5 are permitted to fluctuate within established bounds: for instance, the coefficients for X_1, X_3 and X_5 may oscillate between 600 and 850, 684.668 and 815.332 and 551.373 and 625.553 respectively, whereas X_2 and X_4 possess no lower limits on their respective coefficients but have 571.78 and 536.499 has their upper limits respectively. This degree of flexibility suggests that certain variables hold greater significance, as deviations beyond these thresholds would directly impact the return maximization model.

Moreover, the RHS values of constraints reveal permissible ranges. For instance, the RHS of Constraint 1 is allowed to vary from 7,400 to 10,015.438, while Constraint 2 permits a variation range from 16.651 to 19.00. Similarly, Constraint 3 is capable of varying between 36 and 40.736 units whiles constraint 4 can also vary from 3234.628 with no upper limit. This analysis substantiates the robustness of the solution within these specified ranges, thereby providing valuable insights into potential adjustments in resource allocation without jeopardizing the solution to the return maximization model.

4.1.5: Minimize labour cost using the espilon (ε) constraint method

The formulated model for labour cost minimization was used and in solving it, only the objective for labour cost was considered while the other returns maximization objective was treated as a constraint together with the other constraints. Based on the collected data, the resulting model is as follows:

Minimize $220X_1 + 173.33X_2 + 360X_3 + 200X_4 + 133.33X_5$

Subject to;

$$650X_1 + 366.67X_2 + 500X_3 + 325X_4 + 400X_5 \geq 8500$$

$$X_1 + X_2 + X_3 + X_4 + X_5 \geq 18$$

$$2X_1 + 2X_2 + 3X_3 + 2X_4 + 2X_5 \geq 38$$

$$811.67X_1 + 516.67X_2 + 700X_3 + 400X_4 + 600X_5 \geq 11560$$

$$X_1, X_2, X_3, X_4, X_5 \geq 0$$

Table 4.7 gives the optimal solution for model 3 for vegetable production

Table 4.7: Optimal solution for model 3 for vegetable production

Decision Variables	Optimal acres of land for cultivation	Optimal Labour Cost (in GH¢)
X_1	4.00	
X_2	0.00	
X_3	2.00	3,234.63
X_4	0.00	
X_5	12.00	
Total land in acres	**18**	

From Table 4.7, it is seen that the optimal solution is GH¢ 3,234.63 and proposes production of 4 acres of pepper, 2 acres of onion and 12 acres of tomatoes. It again proposes no production of okro and garden eggs.

Table 4.8 gives the comparism of the existing and the optimal cropping patterns (together with their associated labour cost) for vegetables.

Table 4.8: Comparison of existing and optimal cropping patterns (together with their associated labour cost) for vegetables.

Crops	Existing cropping Plan Size of farm (in acres)	Optimal Cropping plan Size of Farm (in acres)
Pepper	6.00	4.00
Okro	3.00	0.00
Onion	2.00	2.00
Garden eggs	4.00	0.00
Tomatoes	3.00	12.00
Total acres	18.00	18.00
Total Returns (in GH¢)	**3,760.00**	**3,234.63**

From Table 4.8, it is observed that, for a vegetable farmer interested in minimizing cost of labour only, he has to decrease the land used to grow pepper by 2 acres, maintain the land used in cultivating onions and increase that of tomatoes by 9 acres. Also, the difference in cost of labour is $3760.00 - 3234.63 = 525.37$ which indicates approximately 14% decrease in the existing labour cost.

4.1.6 Sensitivity analysis on model 3 for vegetables farming

The sensitivity analysis conducted for the vegetable labour cost minimization model indicates that various coefficients associated with the objective function and the corresponding right-hand-side (RHS) constraint values can be modified within specified limits without undermining the optimal solution.

In particular, the cost coefficients pertaining to decision variables X_1 through X_5 are permitted to fluctuate within established bounds: for instance, the coefficient for X_1 may oscillate between 133.33 and 700.005 whiles that of X_5 has no lower limit but an upper limit 178.820, whereas X_2, X_3 and X_4 possess no upper limits on their respective coefficients but have respective lower limits of 121.775, 167.998 and 107.329. This degree of flexibility suggests that certain variables hold greater significance, as deviations beyond these thresholds would directly impact the minimized labor cost.

Moreover, the RHS values of constraints reveal permissible ranges. For instance, the RHS of Constraint 1 is allowed to vary from 8,061.407 to 11,400, while Constraint 2 permits a variation range from 16.39 to 19.00. Similarly, Constraint 3 is capable of varying between 36 and 49 units whiles that of constraint 4 has no lower limit but an upper limit of 11,931.348. This analysis substantiates the robustness of the solution within these specified ranges, thereby providing valuable insights into potential adjustments in resource allocation without jeopardizing the minimum labour cost solution.

4.2: Result for major cereals and leguminous farming

The second set of farmers were selected based on the major cereals and legumes grown in the study area for commercial purposes. These five major crops grown are rice, maize, millet, cowpea and groundnut. Table 1 gives the five major crops, land area (in acres), number of labourers, labour cost (in GH¢), capital (in GH¢) and returns (in GH¢).

Table 4.9: Overview of major cereals and legumes production

S/No.	Crops	Land Area (in acres)	Number of labourers	Labour Cost (in GH¢)	Capital (in GH¢)	Returns (in GH¢)
1.	Rice	3	9	600	1400	6160
2.	Maize	3	6	420	1000	4840
3.	Groundnut	4	8	400	1200	4400
4.	Millet	2	5	600	900	1400
5.	Cowpea	1	4	350	550	600
	TOTAL	**13**	**32**	**2370**	**5050**	**17400**

Source: Field Survey, 2024

Based on the collected data Table 4.10 which gives the major crops, land area, number of labourers, labour cost, capital and returns per acre was obtained as follows:

Table 4.10: Major cereals and legumes per acre

S/No.	Crops	Land Area (in acres)	Number of labourers per acre	Labour Cost (in GH¢) per acre	Capital (in GH¢) per acre	Returns (in GH¢) per acre
1.	Rice	1	3	200	466.67	2053.33
2.	Maize	1	2	140	333.33	1613.33
3.	Groundnut	1	2	100	300	1100
4.	Millet	1	3	300	450	700
5.	Cowpea	1	4	350	550	600

4.2.1: Maximizing returns and minimizing labour cost using the weighted sum

For Model 1, using a scalarisation weight of 0.5 for returns and 0.5 for labour cost and implementing in the Management Scientist v6.0, the results are as follow;

Maximize $926.67X_6 + 736.67X_7 + 500X_8 + 200X_9 + 125X_{10}$

Subject to;

$$466.67X_6 + 333.33X_7 + 300X_8 + 450X_9 + 550X_{10} \leq 5050$$

$$X_6 + X_7 + X_8 + X_9 + X_{10} \leq 13$$

$$3X_6 + 2X_7 + 2X_8 + 3X_9 + 4X_{10} \leq 32$$

$$X_6, X_7, X_8, X_9, X_{10} \geq 0$$

where X_6 is land area for rice production in acres, X_7 is the land area for maize production in acre X_8 is the land area for groundnut production in acres, X_9 is the land area for millet production in acres and X_{10} is land area under cowpea production in acres.

Table 4.11: Optimal solution for model 1 of the cereals and legumes farming

Decision Variables	Optimal acres of land for cultivation	Optimal returns (in GH¢)
X_6	5.38	
X_7	7.62	
X_8	0.00	10597.97
X_9	0.00	
X_{10}	0.00	
Total land in acres	**13.00**	

From Table 4.11, it is seen that the optimal solution is GH¢ 10,597.97 and proposes production of 5.38 acres of rice and 7.62 acres of maize. It again proposes no production of millet, groundnut and cowpea.

Table 4.12 gives the comparism of the existing and optimal cropping patterns (together with their associated returns).

Table 4.12: Comparison of existing and optimal cropping patterns (together with their associated returns)

Crops	Existing cropping Plan Size of farm (in acres)	Optimal Cropping plan Size of Farm (in acres)
Rice	3.00	5.38
Maize	3.00	7.62
Groundnut	4.00	0.00
Millet	2.00	0.00
Cowpea	1.00	0.00
Total acres	13.00	13.00
Total Returns (in GH¢)	**7,715**	**10,597.97**

From Table 4.12, it is observed that, for a farmer using the existing cropping pattern to make 50% increase in his returns and concurrently decrease his labour cost by 50%, he has to increase the land used to grow rice and maize by 2.38 and 4.62 acres respectively. Also, the difference in returns is $10597.97 - 7515.00 = 3082.97$ which indicates approximately 41% increase in the existing returns.

4.2.2 Sensitivity analysis on model 1 for cereals farming

The sensitivity analysis conducted for the weighted sum cereals model indicates that various coefficients associated with the objective function and the corresponding right-hand-side (RHS) constraint values can be modified within specified limits without undermining the optimal solution.

In particular, the return coefficients pertaining to decision variables X_6 through X_{10} are permitted to fluctuate within established bounds: for instance, the coefficient for X_6 may oscillate between 736.670 and 1,031.356 whiles the coefficients for X_7 also oscillates between 661.896 and 926.670, whereas X_8, X_9 and X_{10} possess no lower limits on their respective coefficients, their upper limits are 689.177, 902.916 and 1,045.409 respectively.

Moreover, the RHS values of constraints reveal permissible ranges. For instance, the RHS of Constraint 1 is allowed to vary from 4,333.290 to 5,133.330, while Constraint 2 permits a variation range from 11.75 to 15.15. Similarly, Constraint 3 is capable of varying from a lower limit of 36 with no upper limits. This analysis substantiates the robustness of the solution within these specified ranges, thereby providing valuable insights into potential adjustments in resource allocation without jeopardizing the solution for the weighted sum model that maximize returns and minimize labour cost.

4.2.3: Maximizing returns using the espilon (ε) constraint method

The formulated model for maximization of returns was used and in solving it, only the objective for returns was considered while the labour cost minimization objective was treated as a constraint together with the other constraints. Based on the collected data, the resulting model is as follows;

Maximize $2053.33X_6 + 1603.33X_7 + 1100X_8 + 700X_9 + 600X_{10}$

Subject to;

$$200X_6 + 140X_7 + 100X_8 + 300X_9 + 350X_{10} \leq 2370$$

$$X_6 + X_7 + X_8 + X_9 + X_{10} \leq 13$$

$$3X_6 + 2X_7 + 2X_8 + 3X_9 + 4X_{10} \leq 32$$

$$466.67X_6 + 333.33X_7 + 300X_8 + 450X_9 + 550X_{10} \leq 5050$$

$$X_6, X_7, X_8, X_9, X_{10} \geq 0$$

Table 4.13 gives the optimal solution of model 2.

Table 4.13: Optimal solution for model 2 for the cereals and legumes farming

Decision Variables	Optimal acres of land for cultivation	Optimal returns (in GH¢)
X_6	5.38	
X_7	7.62	
X_8	0.00	23,338.32
X_9	0.00	
X_{10}	0.00	
Total land in acres	**13.00**	

From Table 4.13, it is seen that the optimal solution is GH¢ 23,338.32 and proposes production of 5.38 acres of rice and 7.62 acres of maize. It again proposes no production of millet, groundnut and cowpea.

Table 4.14 gives the comparison of the existing and optimal cropping patterns (together with their associated returns).

Table 4.14: Comparison of existing and optimal cropping patterns (together with their associated returns)

Crops	Existing cropping Plan Size of farm (in acres)	Optimal Cropping plan Size of Farm (in acres)
Rice	3.00	5.38
Maize	3.00	7.62
Groundnut	4.00	0.00
Millet	2.00	0.00
Cowpea	1.00	0.00
Total acres	13.00	13.00
Total Returns (in GH¢)	**17,400.00**	**23,338.32**

From Table 4.14, it is observed that, for a farmer interested in maximizing returns only, he has to increase the land used to grow rice and maize by 2.38 and 4.62 acres respectively. Also, the difference in returns is $23338.32 - 17400.00 = 5938.32$ which indicates approximately 34% increase in the existing returns.

4.2.4 Sensitivity analysis on model 2 for cereals and legumes farming

The sensitivity analysis conducted on the linear programming model aimed at optimizing cereal returns reveals an exceptionally stable optimal solution. More specifically, the coefficients of the objective function pertaining to variables X_6, X_7 are permitted to fluctuate within the ranges of 1603.33 to 2244.7 and 1466.639 to 2053.33, respectively, without impacting the optimal solution. Conversely, the variables X_8, X_9, and X_{10} exhibit no constraints on their lower bounds, possessing allowable upper limits of 1490.847, 1997.072, and 2334.555, respectively.

In terms of the constraints, the acceptable range for the right-hand side (RHS) of Constraint 1 extends from 2142.503 to infinity, while the permissible variation for Constraint 2 spans from 4333.29 to 5133.33. Furthermore, Constraints 3 and 4 facilitate ranges of 11.75 to 15.15 and from 31.375 to infinity, respectively. These outcomes underscore the model's robustness against variations in return coefficients and resource availability, indicating that stable planning can be achieved under the defined parameter fluctuations.

4.2.5: Minimize labour cost using the espilon (ε) constraint method

The formulated model for labour cost minimization was used and in solving it, only the objective for labour cost was considered while the other returns maximization objective was treated as a constraint together with the other constraints. Based on the collected data, the resulting model is as follows:

Minimize $200X_6 + 140X_7 + 100X_8 + 300X_9 + 350X_{10}$

Subject to;

$$2053.33X_6 + 1603.33X_7 + 1100X_8 + 700X_9 + 600X_{10} \geq 17400$$

$$466.67X_6 + 333.33X_7 + 300X_8 + 450X_9 + 550X_{10} \geq 5050$$

$$X_6 + X_7 + X_8 + X_9 + X_{10} \geq 13$$

$$3X_6 + 2X_7 + 2X_8 + 3X_9 + 4X_{10} \geq 32$$

$$X_6,\ X_7,\ X_8,\ X_9,\ X_{10} \geq 0$$

Table 4.15 gives the optimal solution for model 3

Table 4.15: Optimal solution for model 3 of the cereals and legumes farming

Decision Variables	Optimal acres of land for cultivation	Optimal labour cost (in GH¢)
X_6	6.90	
X_7	0.00	
X_8	6.10	1989.99
X_9	0.00	
X_{10}	0.00	
Total land in acres	13.00	

From Table 4.15, it is seen that the optimal solution is GH¢ 1,989.99 and proposes production of 6.90 acres of rice and 6.10 acres of millet. It again proposes no production of maize, groundnut and cowpea.

Table 4.16 gives the comparism of the existing and the optimal cropping patterns (together with their associated labour cost).

Table 4.16: Comparison of existing and optimal cropping patterns (together with their associated labour cost)

Crops	Existing cropping Plan Size of farm (in acres)	Optimal Cropping plan Size of Farm (in acres)
Rice	3.00	6.90
Maize	3.00	0.00
Groundnut	4.00	6.10
Millet	2.00	0.00
Cowpea	1.00	0.00
Total acres	13.00	13.00
Total Returns (in GH¢)	**2370.00**	**1,989.99**

From Table 4.16, it is observed that, for a farmer interested in minimizing cost of labour only, he has to increase the land used to grow rice and millet by 3.90 acres and that of groundnut by 2.10 acres. Also, the difference in cost of labour is $2370.00 - 1989.99 = 380.01$ which indicates approximately 16% decrease in the existing labour cost.

4.2.6 Sensitivity analysis on model 3 for cereals and legumes farming

The sensitivity analysis conducted for the cereal labour cost minimization model indicates that various coefficients associated with the objective function and the corresponding right-hand-side (RHS) constraint values can be modified within specified limits without undermining the optimal solution.

In particular, the cost coefficients pertaining to decision variables X_6 through X_{10} are permitted to fluctuate within established bounds: for instance, the coefficient for X_6 may oscillate between 100 and 266.670 whiles that of X_8 ranges between 0 to 125.002, whereas X_7, X_9 and X_{10} possess no upper limits on their respective coefficients but have respective lower limits of 119.998, 189.998 and 249.997. This degree of flexibility suggests that certain variables hold greater significance, as deviations beyond these thresholds would directly impact the minimized labour cost.

Moreover, the RHS values of constraints reveal permissible ranges. For instance, the RHS of Constraint 1 is allowed to vary with no lower limit to 11,400, while Constraint 2 permits a variation range from 10.821 to 16.833. Similarly, Constraint 4 is capable of varying between 4,900.020 and 6,066.71 units whiles that of constraint 3 has no lower limit but an upper limit of 32.9. This analysis substantiates the robustness of the solution within these specified ranges, thereby providing valuable insights into potential adjustments in resource allocation without jeopardizing the minimum labour cost solution.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

This chapter presents the summary of the findings from the study and the conclusions made from the findings. It presents the summary on both the vegetables, cereals and legumes results analyzed with the developed multi-objective programming model which sought to help the selected small-scale farmers to maximize the returns and minimize the labour cost concurrently subject to the available land area for cultivation, required total capital and the required number of labours. Also, conclusions and recommendations were made based on the results of this study.

5.2 Summary of findings

This study has proposed a more suitable mathematical models to guide vegetables, cereals and legumes small scale farmers in the Navrongo Municipality of the Upper East Region in Ghana to make informed decisions in their farming activities. These farmers are always faced with the challenges of which size of farm land, required total capital and the required number of labours to be used in cultivating the number of crops they grow in each farming season. These farmers specifically farm for commercial purposes and for that matter are expected to make optimal use of the available land in other to maximize profit or returns at the end of the farming season while the cost of labour is minimized. The formulated models gave several favourable outcomes when applied to data collected. Details of the summary are in 5.2.1 and 5.2.2.

5.2.1 Summary of findings for the vegetable farmers

After the formulated models 1, 2 and 3 were successfully applied to the collected data on vegetables, the results were summarized as follows;

- The vegetable farming industry has a significant effect on the national economy. The current model suggests that if a farmer wants to achieve 60% increase in his returns and concurrently decrease his labour cost by 40%, then he has to decrease and increase the lands used to grow pepper and tomatoes by 1.60 and 8.60 acres respectively. Also, the farmer must maintain the same land used in cultivating onion.
- For a vegetable farmer interested in maximizing returns only, he has to decrease the land used to grow pepper by 1 acre and increase that of tomatoes by 10 acres and he will see approximately 10% increase in his existing net returns.
- Vegetable farmers who are interested in minimizing cost of labour only, they must decrease the land used to grow pepper by 2 acres, maintain the land used in cultivating onions and increase that of tomatoes by 9 acres and they will see approximately 14% decrease in the existing labour cost.

5.2.2 Summary of findings for cereal and legume farmers

After the formulated models 1, 2 and 3 were successfully applied to the collected data on cereals and legumes, the results were summarized as follows;

- For a farmer using the existing cropping pattern to make 50% increase in his returns and concurrently decrease his labour cost by 50%, he has to increase the land used to grow rice and maize by 2.38 and 4.62 acres respectively.

- Farmers who are interested in maximizing returns only, they must increase the land used to grow rice and maize by 2.38 and 4.62 acres respectively and they will realize approximately 34% increase in their existing returns.
- Farmers who are interested in minimizing cost of labour only, they must increase the land used to grow rice and millet by 3.90 acres and that of groundnut by 2.10 acres and they will realize approximately 16% decrease in his existing labour cost.

5.3 Conclusions

It has been realized from the summary of findings that the formulated model helped the small-scale farmers to maximize returns as well as minimizing the cost of labour whilst proposing optimal use of farm land.

The weighted sum and epsilon constraint models that were formulated and tested on the available data gave an optimal solution. Comparing the optimal solutions for both the weighted sum and epsilon constraint method, it was realized that the epsilon constraint gives better optimal returns than that of the weighted sum. This is because the epsilon-constraint technique is frequently favored in comparison to the weighted sum technique due to its provision of a more methodological and adaptable framework for multi-objective optimization. It circumvents the drawbacks associated with arbitrary weight allocation, more effectively addresses conflicting objectives, and facilitates a more comprehensive investigation of the Pareto front, resulting in more representative and varied solutions.

The sensitivity analysis conducted on the models in all test cases showed the robustness of the models.

5.4 Recommendations

Based on the findings of this study, the following recommendations are made:

1. It is evident from the review of related literature that much work has not been done with multi-objective optimization in the field of agriculture specially for crop farmers. It is recommended that further research should be focused on confirming these results by considering other aspects of farm planning as well.
2. Considering the maximization of farming returns, it is recommended that the government through the Ministry of Agriculture will do well to support more small-scale farmers to expand their farming activities in order to make much profit.
3. The model can be extended to tackle other types of farming, although this study has focused on small-scale vegetables, legumes and cereals farming in the Kasena Nankana East District of Ghana. The model can easily be adapted to suit other forms of farming.
4. Future works can also consider adding other parameters like fertilizers application or type of fertilizer used, soil type, water requirements, etc. to see how the model will work.

REFERENCES

Adekanmbi, O. & Olugbara, O. (2015). Multi-objective optimization of crop-mix planning using generalized differential evolution algorithm. *Journal of Agricultural Science and Technology, 17*, 1103-1114.

Al-Hassan, R. & Poulto, C. (2009). Agriculture and social protection in Ghana. *Future Agricultures Consortium Working Paper No. SP04*.

Al-Hassan, R., Diao, X. & Sur, M. (2009). Agricultural growth and investment options for poverty reduction in northern Ghana (IFPRI Discussion Paper 00862). Washington, DC: International Food Policy Research Institute.

Alves, M. J. & Clímaco, J. (2001). Multiobjective mixed-integer programming. In P. M. Pardalos & C. A. Floudas (Eds.), *Encyclopedia of Optimization, Vol. III* (pp. 466–472). Kluwer Academic Publishers.

Anderson, D. R., Sweeney, D. J. & Williams, T. A. (2004). *The management scientist version 6.0* (2nd ed.). Cengage Learning.

Amos, T. T. (2012). *An analysis of productivity and technical efficiency of smallholder cocoa farmers in Nigeria*. University of Ibadan Press.

Armand, P., & Malivert, C. (1993). Trade-offs in multi-objective programming. *Journal of Optimization Theory and Applications*, *78*(3), 421–435.

Assan, K. J., Caminade, C. & Obeng, F. (2009). Environmental variability and vulnerable livelihoods: Minimizing risk and maximizing opportunities for poverty alleviation. *Department of Agricultural Economics, University of Liverpool, 21*, 403-418.

Barnard, C. S., & Nix, J. S. (1999). *Farm planning and control* (revised ed.). Cambridge University Press.

Bazaraa, M. S., Jarvis, J. J. & Sherali, H. D. (2005). *Linear programming and network flows* (4th ed.). Wiley Publications.

Bazaraa, M. S., Jarvis, J. J., & Sherali, H. D. (2010). *Linear programming and network flows* (5th ed.). Wiley.

Beale, E. M. L. (1968). Mathematical programming in practice. *Sir Isaac Pitman & Sons, Ltd.*

Begam, S., Jain, R., Arora, A. & Marwaha, S. (2023). Multi-objective particle swarm optimization for regional crop planning. *Indian Journal of Agricultural Sciences, 93*(2).

Bertsekas, D. P. (2016). *Nonlinear programming* (3rd ed.). Athena Scientific.

Bhadani, K., Asbjörnsson, G., Hulthén, E., Bengtsson, M., & Evertsson, M. (2019). Comparative study of optimization schemes in mineral processing simulations. *Minerals Engineering, 135*, 49-59. https://doi.org/10.1016/j.mineng.2019.03.002

Boyd, S., & Vandenberghe, L. (2004). *Convex optimization*. Cambridge University Press.

Carissimo, C. & Korecki, M. (2023). Limits of Optimization. *Minds and Machines, 34*, 117 - 137.

Chankong, V. & Haimes, Y. Y. (1983). *Multiobjective decision making: Theory and methodology* (North-Holland Series in System Science and Engineering, Vol. 8). North-Holland.

Chen, D., Batson, R. G. & Dang, Y. (2010). *Applied integer programming: Modelling and solution*. John Wiley & Sons.

Chegari, Badr., Tabaa, M.., Simeu, E.., Moutaouakkil, F.., & Medromi, H.. (2021). Multi-objective optimization of building energy performance and indoor thermal comfort by combining artificial neural networks and metaheuristic algorithms. *Energy and Buildings*.

Chvátal, V. (1983). *Linear programming*. W. H. Freeman and Company.

CIA (Central Intelligence Agency). (2013). The World Factbook: Ghana. https://www.cia.gov/library/publications/the-world-factbook/goes/gh.html (Accessed February 5, 2013).

Cohon, J. L. & Marks, D. H. (1975). A review and evaluation of multiobjective programming techniques. *Water Resources Research, 11*, 208-220.

Conitzer, Vincent., Freeman, Rupert., & Shah, Nisarg. (2016). Fair Public Decision Making. *Proceedings of the 2017 ACM Conference on Economics and Computation*. http://doi.org/10.1145/3033274.3085125

Dantzig, G. B. (1976). Linear programming, past and future. In E. I. Salkovitz (Ed.), *Science technology, and the modern navy* (pp. 85–95). Office of Naval Research.

Dantzig, G. B., & Thapa, M. N. (1997). *Linear programming 1: Introduction*. Springer.

Daze, A. (2007). *Climate change and poverty in Ghana*. Care International.

Deb, K. (2005). Multi-objective optimization. In E. K. Burke & G. Kendall (Eds.), *Search methodologies* (pp. 273-316). Springer.

Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. (2002). *A fast and elitist multiobjective genetic algorithm: NSGA-II*. IEEE Transactions on Evolutionary Computation, 6(2), 182–197. https://doi.org/10.1109/4235.996017

Deb, K. (2001). *Multiobjective optimization using evolutionary algorithms*. John Wiley & Sons.

Dessalegn, R. (1999). *Water resources development in Ethiopia: Issues of sustainability and participation*. Forum for Social Studies.

Diao, X., Hazell, P., Resnick, D. & Thurlow, J. (2007). The role of agriculture in development: Implications for Sub-Saharan Africa (IFPRI Research Report 153). Washington, DC: International Food Policy Research Institute.

Doyle, C. J. (1990). Application of systems theory to farm planning and control: Modeling resource allocation. *The Scottish Agricultural College*.

Ehrgott, M. (2005). *Multicriteria optimization* (2nd ed.). Springer.

FAO. (2000). *Food and Agricultural Organization: Trade year book*. Rome.

Food and Agriculture Organization (FAO). (2014). *The state of food and agriculture: Innovation in family farming*. FAO.

Fan, Z., Li, W., Cai, X., Li, H., Wei, C., Zhang, Q., Deb, K. & Goodman, E. D. (2017). Push and pull search for solving constrained multi-objective optimization problems. *IEEE Transactions on Evolutionary Computation, 21*(3), 367-380.

Fathollahi-Fard, A. M., Hajiaghaei-Keshteli, M., Mirjalili, S. & Mathur, A. (2023). A fuzzy multi-objective optimization model for sustainable harvest planning: Balancing profit, greenhouse gas emissions, and waste. *Journal of Cleaner Production, 386*, 135706.

Garcia, J.A. & Alamanos, A. (2022). Integrated Modelling Approaches for Sustainable Agri-Economic Growth and Environmental Improvement: Examples from Canada, Greece, and Ireland. *SSRN Electronic Journal*.

Gembicki, F. W. (1974). Vector optimization for control with performance and parameter sensitivity indices (Ph.D. thesis). Case Western Reserve University, Cleveland, Ohio.

GFA Consulting Group. (2010). *Rural income and poverty reduction strategies in Africa: Report for the period 1992-2006*. Hamburg, Germany.

Ghana Statistical Service. (2012). *The 2010 population and housing census, summary report of final results*. Accra, Ghana: Ghana Statistical Service.

Ghersa, C. M., Burgos, N. R., Caviglia, O. P. & Oesterheld, M. (2024). The AgrOptim framework: Integrating crop simulation and genetic algorithms to optimize economic and biophysical indicators. *Agricultural Systems, 205*, 103649. https://doi.org/10.1016/j.agsy.2023.103649

Giuseppe, N. (2008). Classical methods for multiobjective optimization. Courant Institute of Mathematical Sciences, New York University.

Gurmesa, N. D. (2011). *Farm optimization for households growing oilseed crop in Jimma, Ethiopia* (MSc thesis). Wageningen University, The Netherlands.

Hazell, P. B. R. & Norton, R. D. (1986). *Mathematical programming for economic analysis in agriculture*. Macmillan Publishing Company.

Heady, E. O., & Dillon, J. L. (1961). *Agricultural production functions*. Iowa State University Press.

Hillier, F. S. & Lieberman, G. J. (2010). *Introduction to operations research* (9th ed.). McGraw-Hill.

Hillier, F. S., & Lieberman, G. J. (2014). *Introduction to operations research* (10th ed.). McGraw-Hill Education.

International Fund for Agricultural Development (2012). *Ghana: Country programme evaluation* (IFAD Publ. No. 84). May 2012. http://www.ifad.org/evaluation/public html/eksyst/doc/profile/pa/ghana2012.htm (Accessed February 19, 2013).

Igwe, K. C. & Onyenweaku, C. E. (2013). A linear programming approach to food crops and livestock enterprises planning in Aba Agricultural Zone of Abia State, Nigeria. *American Journal of Experimental Agriculture, 3*(2), 412-431.

Irz, X., Lin, L., Thirtle, C. & Wiggins, S. (2001). Agricultural growth and poverty alleviation. *Development Policy Review, 19*(4), 449–466.

Jain, S., Ramesh, D. & Bhattacharya, D. (2021). A multi-objective algorithm for crop pattern optimization in agriculture. *Applied Soft Computing, 112*, 107772.

Jurgen, B. K. & Kaisa, M. (2008). *Multiobjective optimization: Interactive and evolutionary approaches*. Springer Publishers.

Kaisa, M. (1999). *Nonlinear multiobjective optimization*. Springer. https://doi.org/10.1007/978-0-7923-8278-2.

Kavzoglu, T.., Şahin, E.., & Colkesen, I.. (2014). Landslide susceptibility mapping using GIS-based multi-criteria decision analysis, support vector machines, and logistic regression. *Landslides* , 11 , 425-439 . http://doi.org/10.1007/s10346-013-0391-7

Kim, I. Y. & de Weck, O. L. (2006). Adaptive weighted sum method for multiobjective optimization: A new method for Pareto front generation. *Structural and Multidisciplinary Optimization, 31*(2), 105-116. https://doi.org/10.1007/s00158-005-0545-3

Konak, D., Coit, W. & Smith, A. E. (2006). Multi-objective optimization using genetic algorithms: A tutorial. *Reliability Engineering & System Safety, 91*(9), 992-1007. https://doi.org/10.1016/j.ress.2005.06.033

Konno, H. & Yamazaki, H. (1991). Mean-absolute deviation portfolio optimization model and its applications to the Tokyo stock market. *Management Science, 37*(4), 519-531. https://doi.org/10.1287/mnsc.37.4.519

Krylovas, A., Kosareva, N. & Zavadskas, E.K. (2017). WEBIRA - Comparative Analysis of Weight Balancing Method. *Int. J. Comput. Commun. Control, 12*, 238-253.

Laufer, B., Gilbert, T.K. & Nissenbaum, H. (2023). Optimization's Neglected Normative Commitments. *Proceedings of the 2023 ACM Conference on Fairness, Accountability, and Transparency*.

Leahi, A. (1988). Irrigation development in Sub-Saharan Africa: Future prospects. In M. Guyle et al. (Eds.), *Developing and improving irrigation and drainage systems* (pp. 3-19). The World Bank.

Lewis, C. (2008). *Linear programming, theory and applications*. Retrieved from https://www.whitman.edu/Documents/Academics/Mathematics/lewis.pdf

Liang, Jing J.., Ban, Xuanxuan., Yu, Kunjie., Qu, B.., Qiao, Kangjia., Yue, Caitong., Chen, Ke., & Tan, K.. (2023). A Survey on Evolutionary Constrained Multiobjective Optimization. *IEEE Transactions on Evolutionary Computation* , 27 , 201-221 . http://doi.org/10.1109/TEVC.2022.3155533

Luenberger, D. G., & Ye, Y. (2016). *Linear and nonlinear programming* (4th ed.). Springer.

My Agricultural Information Bank (MAIB). (2015). Retrieved from http://www.agriinfo.in/default.aspx?page=topic&superid=10&topicid=37

Majeke, F. (2013). Modeling a small farm livelihood system using linear programming in Bindura, Zimbabwe. *Research Journal of Management Sciences, 2*(5), 20-23. ISSN 2319–1171.

Marsh, K., Lanitis, T., Neasham, D., Orfanos, P., & Caro, J. (2014). Assessing the Value of Healthcare Interventions Using Multi-Criteria Decision Analysis: A Review of the Literature. *Pharmaco Economics* , 32, 345-365

Marttunen, M., Lienert, J., & Belton, V. (2017). Structuring problems for Multi-Criteria Decision Analysis in practice: A literature review of method combinations. *Eur. J. Oper. Res.*, 263 ,1-17.

McCarl, B. A. & Spreen, T. H. (2013). *Linear programming modeling*. Springer.

Metcalf, R. L. (1969). *The economics of agriculture*. Harmondsworth: Penguin Books.

Ministry of Agriculture. (2023). Agricultural sector and economic overview: Employment and GDP contributions in Ghana. Accra: Government Printing Press. https://www.moa.gov.gh/reports/agricultural-overview-2023

Mirjalili, Seyed Mohammad. (2018). Evolutionary Algorithms and Neural Networks - Theory and Applications, 780, 3-156.

Mokhtar, M.R., Abdullah, P.B., Hassan, M.Y. & Hussin, F. (2017). Comparative study of Multiple Criteria Decision-Making methods for selecting the best Demand Side Management options. https://api.semanticscholar.org/CorpusID:157489506

Mugabe, D. (2014). Estimation of optimal land use allocation among smallholder (A1) farmer households in Zimbabwe: A case study of Long Croft Farm in Mazowe District. *Journal of Agricultural Science, 6*(2), 113-123.

Nagy, L., Pusztai, L. & Csipkés, M. (2018). Application of the multiple objective programming in the optimization of production structure of an agricultural holding. *Journal of Agricultural Informatics* (ISSN 2061-862X) 2018 Vol. 9, No. 2:54-65

Nocedal, J., & Wright, S. J. (2006). *Numerical optimization* (2nd ed.). Springer.

Osika, Z., Salazar, J. Z., Roijers, D.M., Oliehoek, F. A. & Murukannaiah, P.K. (2023). What Lies beyond the Pareto Front? A Survey on Decision-Support Methods for Multi-Objective Optimization. *ArXiv, abs/2311.11288*.

Otieno, F. & Adeyemo, J. (2011). Multi-objective cropping pattern in Vaalharts irrigation scheme. *African Journal of Agricultural Research, 6*(6), 1286-1294.

Otoo, J., Ofori, J. K. & Amoah, F. (2015). Optimal selection of crops: A case study of small-scale farms in Fanteakwa District, Ghana. *International Journal of Scientific & Technology Research, 4*, 1-8. ISSN 2277-8616.

Paez, J. D., Smith, R., Thompson, L. & Alvarez, G. (2023). A multi-objective optimization framework for strategic drought management: Balancing agricultural and hydrological needs. *Journal of Environmental Management, 320*, 115761.

Radhika, S. & Chaparala, A. (2018). Optimization using evolutionary metaheuristic techniques: a brief review. *Brazilian journal of operations & production management, 15*, 44-53.

Rahimi, I., Gandomi, A.H., Chen, F. & Mezura-Montes, E. (2022). A Review on Constraint Handling Techniques for Population-based Algorithms: from single-objective to multi-objective optimization. *Archives of Computational Methods in Engineering, 30*, 2181-2209.

Randall, P., Singh, R., Zhao, Y. & Martinez, J. (2024). Integrating climate projections into optimization models for sustainable water resource management and crop planning. *Environmental Modelling & Software, 165*, 105612.

Ruzika, S. & Wiecek, M. (2005). Approximation methods in multiobjective programming. *Journal of Optimization Theory and Applications, 126*(3), 473-501. https://doi.org/10.1007/s10957-005-2610-4

Sahoo, S. K.., & Goswami, S. (2023). A Comprehensive Review of Multiple Criteria Decision-Making (MCDM) Methods: Advancements, Applications, and Future Directions. *Decision MakingAdvances*. http://doi.org/10.31181/dma1120237

Shepherd, A. (2005). *The implications of the financial crisis for developing countries*. Institute of Development Studies.

Siksnelyte-Butkiene, Indre., Zavadskas, E., & Štreimikienė, D. (2020). Multi-Criteria Decision-Making (MCDM) for the Assessment of Renewable Energy Technologies in a Household: A Review. *Energies*, 13 , 1-22

Sofi, N. (2015). Decision making in agriculture: A linear programming approach. *International Journal of Modern Mathematical Sciences, 13*(2), 160-169.

Sorooshian, S. & Parsia, Y. (2019). Modified Weighted Sum Method for Decisions with Altered Sources of Information. *Mathematics and Statistics*.

Swanson, E. R. (1956). Application of programming analysis to Corn Belt farms. *Journal of Farm Economics, 38*(2), 408-419. https://doi.org/10.2307/1233309

Tian, Ye., Zhang, Xing-yi., Wang, Chao., & Jin, Yaochu. (2020). An Evolutionary Algorithm for Large-Scale Sparse Multiobjective Optimization Problems. *IEEE Transactions on Evolutionary Computation*, 24, 380-393

Todman, L. C., Coleman, K., Milne, A. E., Gil, D. B., Reidsma, P., Schwoob, M. H., Treyer, S. & Whitmore, P. A. (2019). Multi-objective optimization as a tool to identify possibilities for future agricultural landscapes. *Science of the Total Environment*, *Volume 687*, [Pages 535-545].

Tyler, G. T. (1958). An application of linear programming. *Journal of Agricultural Economics, 13*, 473-485.

United States Agency for International Development (USAID). (2013). Agriculture and food security. Retrieved from http://www.usaid.gov/ghana/agriculture-and-food-security

Van Wijk, T. M., Rufino, M. C., Enahoro, D. K., Parsons, D., Silvestri, S., Valdivia, R. O. & Herrero, T. M. (2012). A review on farm household modelling with a focus on climate change adaptation and mitigation. *Agricultural Systems, 112*, 1-12.

Vaz, F., Lavinas, Y., Aranha, C. D. & Ladeira, M. B. (2020). Exploring Constraint Handling Techniques in Real-world Problems on MOEA/D with Limited Budget of Evaluations. *International Conference on Evolutionary Multi-Criterion Optimization*.

Villareal, B. & Karwan, M. H. (1981). Multicriteria integer programming: A hybrid dynamic programming recursive approach. *Mathematical Programming, 21*, 204–223. https://doi.org/10.1007/BF01586579.

Wang, Handing., Olhofer, M.., & Jin, Yaochu. (2017). A mini-review on preference modeling and articulation in multi-objective optimization: current status and challenges. *Complex & Intelligent Systems*, 3, 233 - 245

Wang, Zhiyuan., & Rangaiah, G. P.. (2017). Application and Analysis of Methods for Selecting an Optimal Solution from the Pareto-Optimal Front obtained by Multiobjective Optimization. *Industrial & Engineering Chemistry Research*, 56 , 560-574. http://doi.org/10.1021/ACS.IECR.6B03453.

Whitmore, P. A. (2019). Multi-objective optimization as a tool to identify possibilities for future agricultural landscapes. *Science of the Total Environment*, *Volume 687*, [Pages 535-545].

World Bank. (1998). *World development report 1998/1999: Knowledge for development*. Oxford University Press.

World Bank. (2010). *Agricultural growth and poverty reduction: Additional evidence*. World Bank.

World Food Programme (WFP). (2009). *Ghana Comprehensive Food Security and Vulnerability Analysis Assessment*. United Nations, Accra Office - Ghana.

Yang, G., Li, X., Huo, L. & Liu, Q. (2020). A solving approach for fuzzy multi-objective linear fractional programming and application to an agricultural planting structure optimization problem. *Chaos, Solitons & Fractals, 141*, 110352.

Zhai, Z., Ortega, J. M., Martínez, N. L. & Rodríguez-Molina, J. (2018). A Mission Planning Approach for Precision Farming Systems Based on Multi-Objective Optimization. *Sensors*, *18*(6), 1795.

Zhang, J., Huang, Yimiao., Wang, Yuhang., & Ma, G. (2020). Multi-objective optimization of concrete mixture proportions using machine learning and metaheuristic algorithms. *Construction and Building Materials*.

Zhang, Q., & Li, H. (2007). MOEA/D: A multiobjective evolutionary algorithm based on decomposition. *IEEE Transactions on Evolutionary Computation*, 11(6), 712–731. https://doi.org/10.1109/TEVC.2007.892759

Zheng, Y., Song, Q. & Chen, S. (2013). Multiobjective fireworks optimization for variable-rate fertilization in oil crop production. *Applied Soft Computing Journal, 13*(8), 3791-3802.